



STUDIECENTRUM VOOR KERNENERGIE  
CENTRE D'ETUDE DE L'ENERGIE NUCLEAIRE

# Efficiency transfer for low-energy (30-100 keV) gamma-ray spectrometry analyses

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# Efficiency transfer for low-energy (30-100 keV) gamma-ray spectrometry analyses

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Low Level Radioactivity Measurements

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- Efficiency transfer for routine gamma-ray analyses
  - Needs
  - Organization at SCK•CEN (automated efficiency transfer)
  - Problems at low-energy
- Material specific correction for gamma-attenuation
  - Generic material descriptions
  - Transmission experiments – element analyses & correction methods
- TEFFTRAN
  - Relative detector response for a transmission experiment
- Efficiency transfer by EFFTRAN with input from TEFFTRAN
  - Proposal procedure

# Efficiency transfer for routine gamma-ray analyses

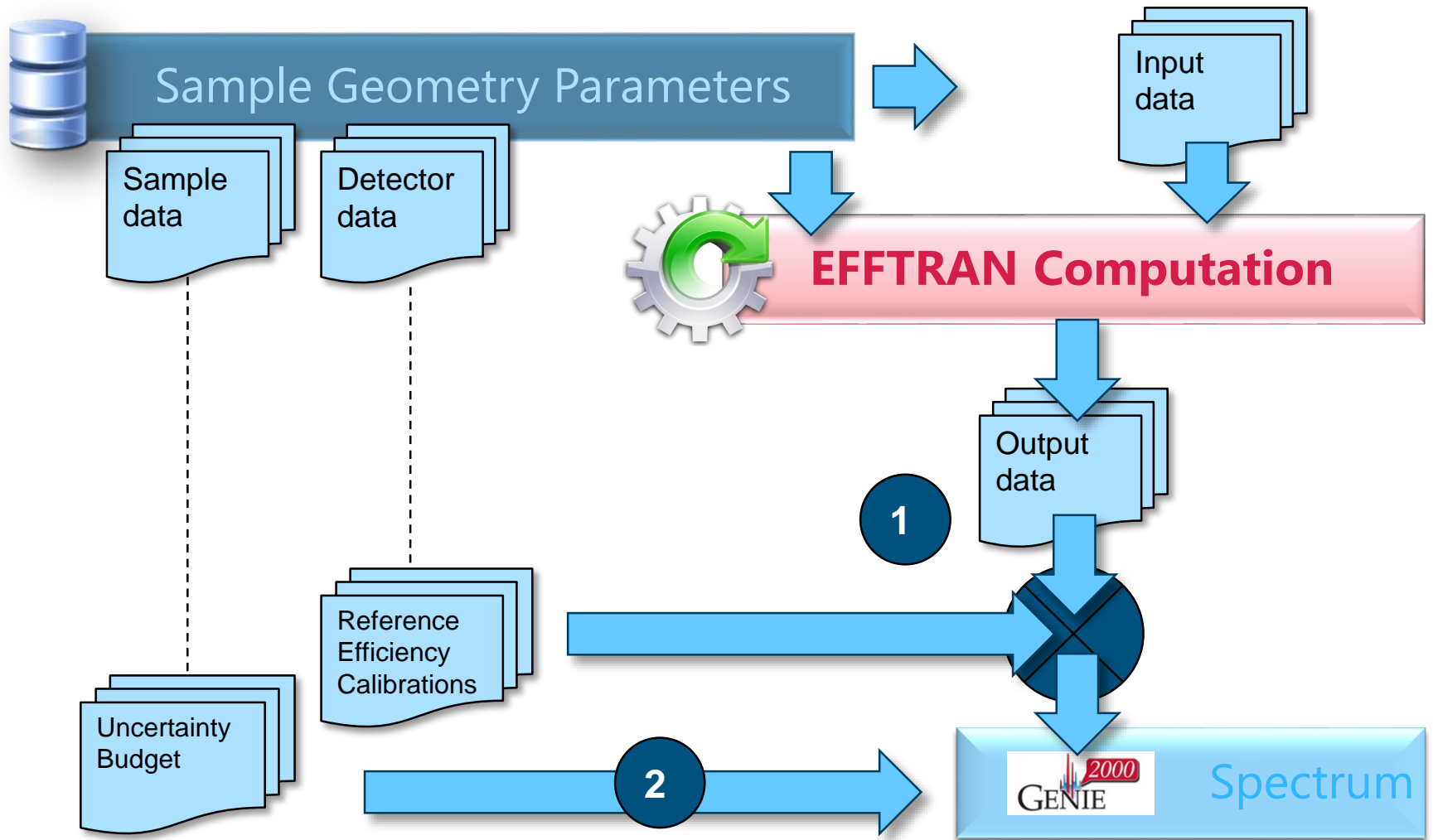
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- Efficiency transfer is required for unbiased results
  - Filling height (if no 100% filling is used)
  - Apparent sample density
  - Sample composition (30-100 keV) 
  - Any other sample parameter that is different from the reference
- Other corrections that may depend on geometry
  - Corrections for ambient background
    - Filling height, density,... may modify background radiation that enters the detector
    - Generally neglected
  - True summing corrections
    - unbiased or small bias when the actual sample parameters are used

- Database with sample data
  - Sample container parameters (dimensions, materials)
  - Sample net weight
  - Sample volume (filling height)
  - Sample composition → material list → attenuation data via XCOM<sup>1</sup>
  - All sample parameters configured according to EFFTRAN
- Database with detector model data (according to EFFTRAN)
- Reference efficiency calibrations
  - Multi gamma sources + additional nuclides
  - Water matrix (acidified)
  - Fixed volume/filling height (per geometry)
  - Stored as Genie 2000 (cal files)
  - Referenced in database (by geometry ID and detector ID)

<sup>1</sup> Photon Cross Sections Database, NIST

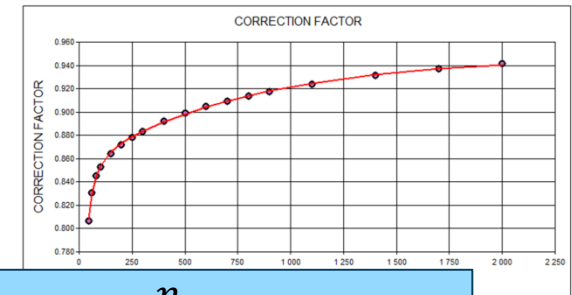
# Efficiency Transfer and Uncertainty Budget



# Efficiency Transfer data model Correction function

## EFFTRAN Computation

Actual Sample ↔ Reference Sample



$$E_1 \rightarrow CF(E_1)$$

$$E_2 \rightarrow CF(E_2)$$

$$E_3 \rightarrow CF(E_3)$$

$$E_4 \rightarrow CF(E_4)$$

...

$$E_n \rightarrow CF(E_n)$$

$$\ln(\mathbf{CF}(\ln(E))) = \sum_{k=0}^n \mathbf{C}_k \ln(E)^k$$

Spectrum (cam file data)

$$\ln \varepsilon_{ref}(E) = \sum_{k=0}^n A_k \ln(E)^k$$

$$\ln \varepsilon_{Corr}(E) = \sum_{k=0}^n (A_k + \mathbf{C}_k) \ln(E)^k$$

$$\varepsilon_{Sam} = \varepsilon_{ref} \mathbf{CF}$$

Applied to low and high-energy dual efficiency curve

# Uncertainty Budget (function of gamma-ray energy)



Sample type → Uncertainty Polynomial

$$Err(\ln(E)) = \sum_{k=0}^n E_k \ln(E)^k$$

Spectrum (cam file data)

Covariance matrix of polynomial fitting + source uncertainty

$$\sigma^2(\ln \varepsilon(\ln(E))) = \sum_{j=0}^n \sum_{i=0}^n M_{ij} \ln(E)^{i+j}$$

$$\sigma^2(\ln(\varepsilon)) = \frac{\sigma^2(\varepsilon)}{\varepsilon^2}$$

Efficiency function for standard uncertainty used by Genie

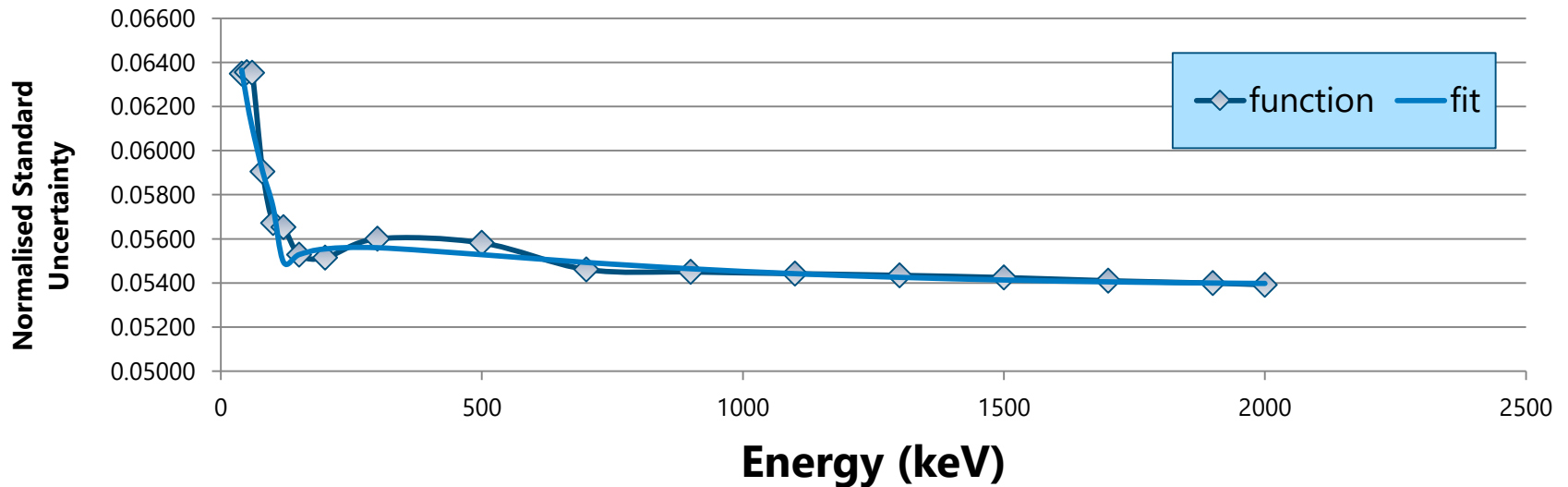
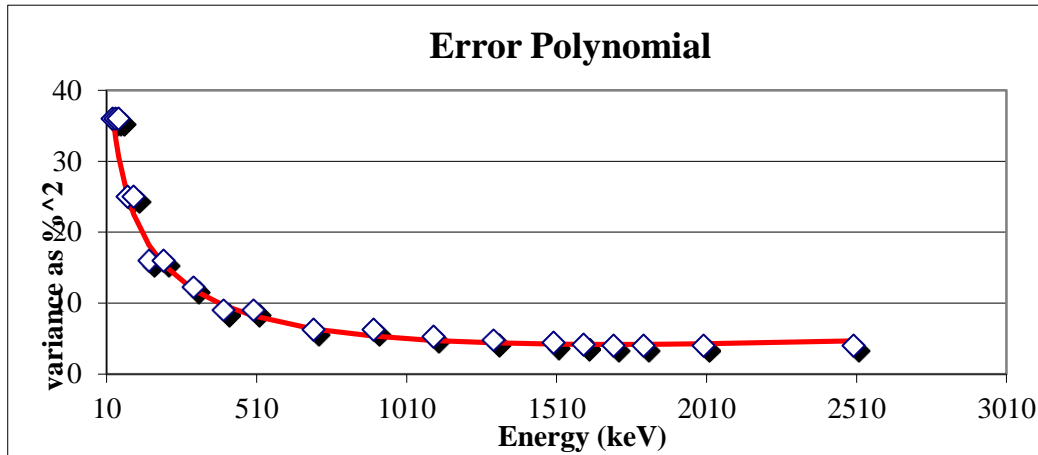
Applied to low and high-energy data



# Determination of energy depended uncertainty budget

| REF | waar ENERGY(keV) | VULHOOGTE |        |       | DIAMETER |        |        | BODEM DIKTE |        |       | SAMPLE DENSITY |        |        | Sample COMPOSITION |        |        | POSITIONERING STAAL | CURVE FITTING | REF KALIBRATIEBRON | SYSTEEM DRIFT | GAMMA INTENSITEIT | SUMMING CORRECTION | TELVERLIEZEN | ENERGY (keV) | STANDARD COMBINED UNCERTAINTY | VARIANCE |
|-----|------------------|-----------|--------|-------|----------|--------|--------|-------------|--------|-------|----------------|--------|--------|--------------------|--------|--------|---------------------|---------------|--------------------|---------------|-------------------|--------------------|--------------|--------------|-------------------------------|----------|
|     |                  | -3%       | 3%     | MAX   | -0.2%    | 0.2%   | MAX    | -2%         | 2%     | MAX   | -3%            | 3%     | MAX    | A                  | B      | MAX    | MAX                 | 0.02<br>k=1   | 0.01               | 0<br>bib      | 0.02              | 0.01               | 45           |              | Variance                      |          |
| 1   | 45               | -0.0537   | 0.0489 | 0.031 | -0.0016  | 0.0009 | 0.0009 | -0.0018     | 0.0011 | 0.001 | -0.0239        | 0.0177 | 0.0138 |                    | 0.4092 | 0.0680 | 0.02                | 0.02          | 0.01               | 0             | 0.02              | 0.01               | 45           | 0.419        | 0.1754                        |          |
| 2   | 60               | -0.0534   | 0.0481 | 0.031 | -0.0008  | 0.0016 | 0.0009 | -0.0016     | 0.0016 | 0.001 | -0.0213        | 0.0162 | 0.0123 |                    | 0.2333 | 0.0650 | 0.02                | 0.02          | 0.01               | 0             | 0.02              | 0.01               | 60           | 0.249        | 0.0619                        |          |
| 3   | 80               | -0.0526   | 0.0474 | 0.030 | -0.0010  | 0.0015 | 0.0009 | -0.0015     | 0.0015 | 0.001 | -0.0201        | 0.0153 | 0.0116 |                    | 0.1261 | 0.0610 | 0.02                | 0.02          | 0.01               | 0             | 0.02              | 0.01               | 80           | 0.151        | 0.0228                        |          |
| 4   | 100              | -0.0521   | 0.0474 | 0.030 | -0.0009  | 0.0013 | 0.0007 | -0.0017     | 0.0017 | 0.001 | -0.0188        | 0.0147 | 0.0108 |                    | 0.1248 | 0.0590 | 0.02                | 0.02          | 0.01               | 0             | 0.02              | 0.01               | 100          | 0.149        | 0.0221                        |          |
| 5   | 150              | -0.0510   | 0.0467 | 0.029 | -0.0012  | 0.0012 | 0.0007 | -0.0020     | 0.0012 | 0.001 | -0.0170        | 0.0139 | 0.0099 |                    | 0.0419 | 0.0540 | 0.02                | 0.02          | 0.01               | 0             | 0.02              | 0.01               | 150          | 0.087        | 0.0076                        |          |
| 6   | 200              | -0.0501   | 0.0458 | 0.029 | -0.0008  | 0.0012 | 0.0007 | -0.0016     | 0.0016 | 0.001 | -0.0164        | 0.0133 | 0.0095 |                    | 0.0307 | 0.0500 | 0.02                | 0.02          | 0.01               | 0             | 0.02              | 0.01               | 200          | 0.080        | 0.0063                        |          |
| 5   | 150              | -0.0510   | 0.0467 | 0.029 | -0.0012  | 0.0012 | 0.0007 | -0.0020     | 0.0012 | 0.001 | -0.0170        | 0.0139 | 0.0099 |                    | 0.0419 | 0.0540 | 0.02                | 0.02          | 0.01               | 0             | 0.02              | 0.01               | 150          | 0.080        | 0.0063                        |          |
| 6   | 200              | -0.0501   | 0.0458 | 0.029 | -0.0008  | 0.0012 | 0.0007 | -0.0016     | 0.0016 | 0.001 | -0.0164        | 0.0133 | 0.0095 |                    | 0.0307 | 0.0500 | 0.02                | 0.02          | 0.01               | 0             | 0.02              | 0.01               | 200          | 0.080        | 0.0063                        |          |
| 7   | 250              | -0.0492   | 0.0448 | 0.028 | -0.0008  | 0.0012 | 0.0007 | -0.0016     | 0.0016 | 0.001 | -0.0159        | 0.0122 | 0.0092 |                    | 0.0294 | 0.0480 | 0.03                | 0.01          | 0.01               | 0             | 0.02              | 0.01               | 250          | 0.084        | 0.0070                        |          |
| 8   | 300              | -0.0484   | 0.0443 | 0.028 | -0.0004  | 0.0012 | 0.0007 | -0.0017     | 0.0012 | 0.001 | -0.0154        | 0.0117 | 0.0089 |                    | 0.0286 | 0.0460 | 0.02                | 0.01          | 0.01               | 0             | 0.02              | 0.01               | 300          | 0.076        | 0.0057                        |          |
| 9   | 400              | -0.0474   | 0.0430 | 0.027 | -0.0004  | 0.0013 | 0.0007 | -0.0017     | 0.0013 | 0.001 | -0.0138        | 0.0117 | 0.0080 |                    | 0.0266 | 0.0420 | 0.02                | 0.02          | 0.01               | 0             | 0.02              | 0.01               | 400          | 0.072        | 0.0052                        |          |
| 10  | 500              | -0.0463   | 0.0424 | 0.027 | -0.0009  | 0.0009 | 0.0005 | -0.0017     | 0.0009 | 0.001 | -0.0133        | 0.0106 | 0.0077 |                    | 0.0261 | 0.0390 | 0.02                | 0.02          | 0.01               | 0             | 0.02              | 0.01               | 500          | 0.070        | 0.0049                        |          |
| 11  | 600              | -0.0452   | 0.0416 | 0.026 | -0.0004  | 0.0013 | 0.0008 | -0.0013     | 0.0013 | 0.001 | -0.0122        | 0.0101 | 0.0071 |                    | 0.0247 | 0.0370 | 0.01                | 0.01          | 0.01               | 0             | 0.02              | 0.01               | 600          | 0.063        | 0.0040                        |          |
| 12  | 700              | -0.0447   | 0.0411 | 0.026 | -0.0004  | 0.0013 | 0.0008 | -0.0013     | 0.0013 | 0.001 | -0.0117        | 0.0096 | 0.0068 |                    | 0.0232 | 0.0350 | 0.01                | 0.01          | 0.01               | 0             | 0.02              | 0.01               | 700          | 0.062        | 0.0038                        |          |
| 13  | 800              | -0.0443   | 0.0403 | 0.026 | -0.0004  | 0.0013 | 0.0008 | -0.0013     | 0.0013 | 0.001 | -0.0112        | 0.0096 | 0.0065 |                    | 0.0226 | 0.0340 | 0.01                | 0.01          | 0.01               | 0             | 0.02              | 0.01               | 800          | 0.061        | 0.0037                        |          |
| 14  | 900              | -0.0435   | 0.0401 | 0.025 | -0.0009  | 0.0009 | 0.0005 | -0.0018     | 0.0009 | 0.001 | -0.0107        | 0.0091 | 0.0062 |                    | 0.0216 | 0.0320 | 0.01                | 0.01          | 0.01               | 0             | 0.02              | 0.01               | 900          | 0.059        | 0.0035                        |          |
| 15  | 1100             | -0.0426   | 0.0394 | 0.025 | -0.0005  | 0.0009 | 0.0005 | -0.0014     | 0.0009 | 0.001 | -0.0103        | 0.0082 | 0.0060 |                    | 0.0199 | 0.0300 | 0.01                | 0.01          | 0.01               | 0             | 0.02              | 0.01               | 1100         | 0.057        | 0.0033                        |          |
| 16  | 1400             | -0.0418   | 0.0382 | 0.024 | -0.0005  | 0.0010 | 0.0006 | -0.0014     | 0.0010 | 0.001 | -0.0089        | 0.0078 | 0.0051 |                    | 0.0173 | 0.0270 | 0.01                | 0.01          | 0.01               | 0             | 0.02              | 0.01               | 1400         | 0.054        | 0.0030                        |          |
| 17  | 1700             | -0.0409   | 0.0375 | 0.024 | -0.0005  | 0.0010 | 0.0006 | -0.0010     | 0.0010 | 0.001 | -0.0079        | 0.0074 | 0.0046 |                    | 0.0153 | 0.0250 | 0.01                | 0.01          | 0.01               | 0             | 0.02              | 0.01               | 1700         | 0.053        | 0.0028                        |          |
| 18  | 2000             | -0.0399   | 0.0371 | 0.023 | -0.0005  | 0.0010 | 0.0006 | -0.0010     | 0.0010 | 0.001 | -0.0081        | 0.0063 | 0.0047 |                    | 0.0137 | 0.0230 | 0.01                | 0.01          | 0.01               | 0             | 0.02              | 0.01               | 2000         | 0.051        | 0.0026                        |          |

# Uncertainty function and corresponding polynomial



# An uncertainty budget function is defined for each counting geometry

ditor

Geometry 20mL Scintillatie PE

20mL Scintillatie PE

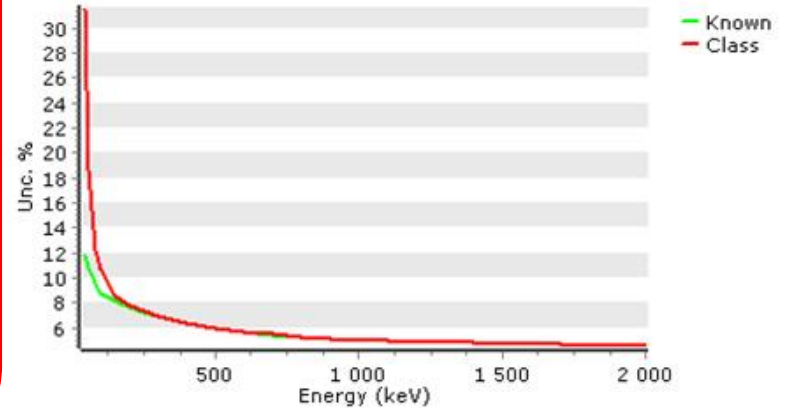
## ERROR PARAMETERS\*

Show

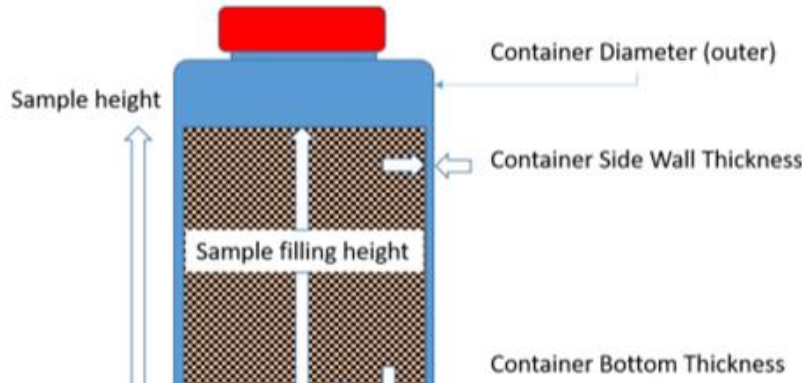
\*The error parameters define a polynomial  $Unc(Energy)$  that corresponds to the expanded uncertainty, excluding: counting statistics, background correction, uncertainty on intensity

|        | LOW                   | HIGH                  |                       |
|--------|-----------------------|-----------------------|-----------------------|
| ERR_0: | -2.64190897668886     | -0.217495281069351    | K<br>N<br>O<br>W<br>N |
| ERR_1: | 2.40934617013547      | 0.153041143179349     |                       |
| ERR_2: | -0.809828432839497    | -3.77714915855748E-02 |                       |
| ERR_3: | 0.119617242483509     | 4.0037761660443E-03   |                       |
| ERR_4: | -6.56724737353403E-03 | -1.55692968480513E-04 |                       |
| ERR_0: | 46.2408477481583      | -0.212328033862284    | C<br>L<br>A<br>S<br>S |
| ERR_1: | -38.4042839799321     | 0.151965311319783     |                       |
| ERR_2: | 11.9564513751801      | -3.79126312099868E-02 |                       |
| ERR_3: | -1.65299820997481     | 4.05003978683371E-03  |                       |
| ERR_4: | 8.56024431411862E-02  | -1.58424173065154E-04 |                       |

ERROR FUNCTIONS (k=1)



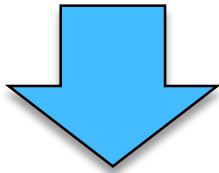
|       |
|-------|
| 41.05 |
| 1.03  |
| 26.94 |
| 1.25  |
| 1.03  |
| 0.941 |
| 0     |



# Generic sample compositions for sample matrices

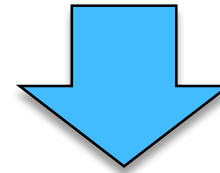
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- well defined materials (water, PE, metals...)



Uncertainty budget does not assume variability of the sample composition

- material class
  - Organic matter
  - Soil/dirt




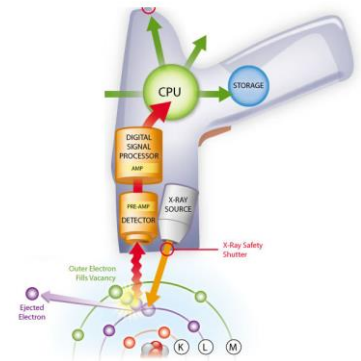
Uncertainty budget **accounts for variability of the sample composition** and its impact on detection efficiency

Uncertainty gets unrealistically large at low energy !



# Means to determine sample attenuation

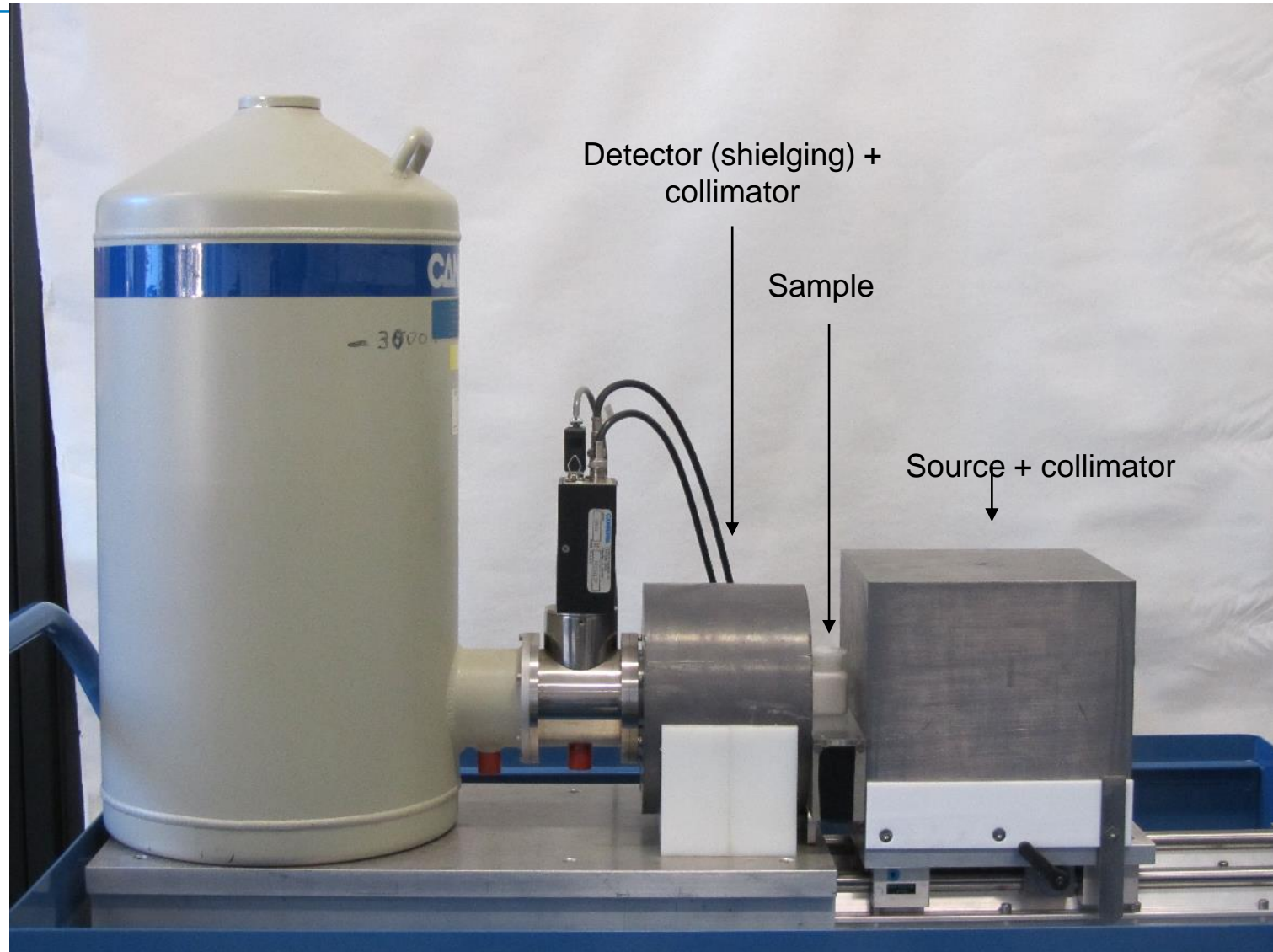
- Generic compositions 
- By comparison with representative standards
  - (not very practical for general use)
- XRF-handheld monitor, AOS-MS, ...
  - Specify the sample material for the EFFTRAN sample model
    - Elements from Mg → U, other elements O and H ?
    - Apply a standard efficiency transfer with EFFTRAN
      - Attenuation data from XCOM
- Multi Energy gamma-ray transmission measurement through sample material



$$T(E) = \exp(-\mu_l(E) x)$$

Only valid for parallel beams and well defined geometry ?

# Transmission setup at SCK-CEN



# Experimental determination of linear attenuation coefficient

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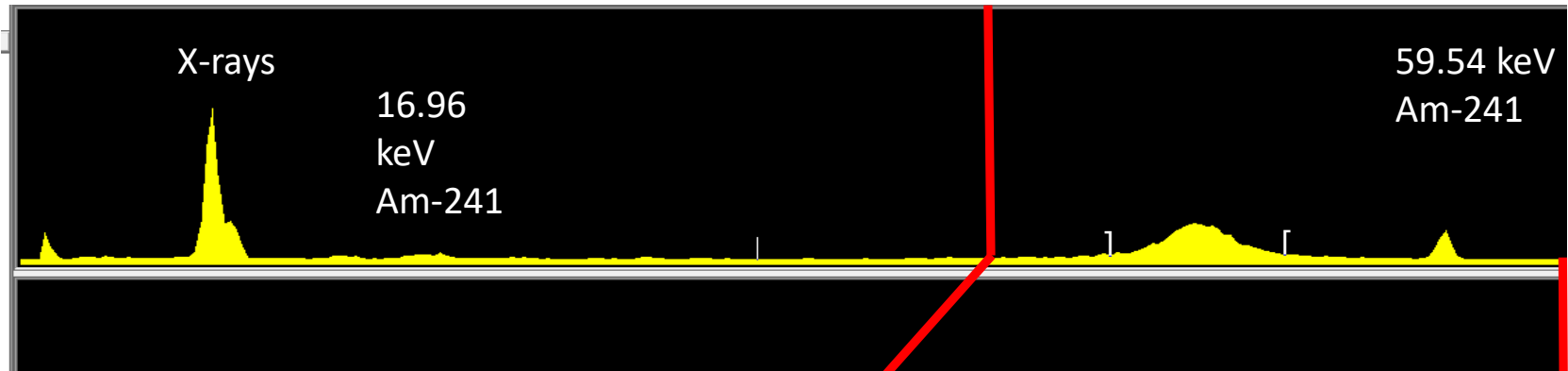
- Well collimated beam is required

$$R(x) = R_0 \exp(-\mu_l x)$$

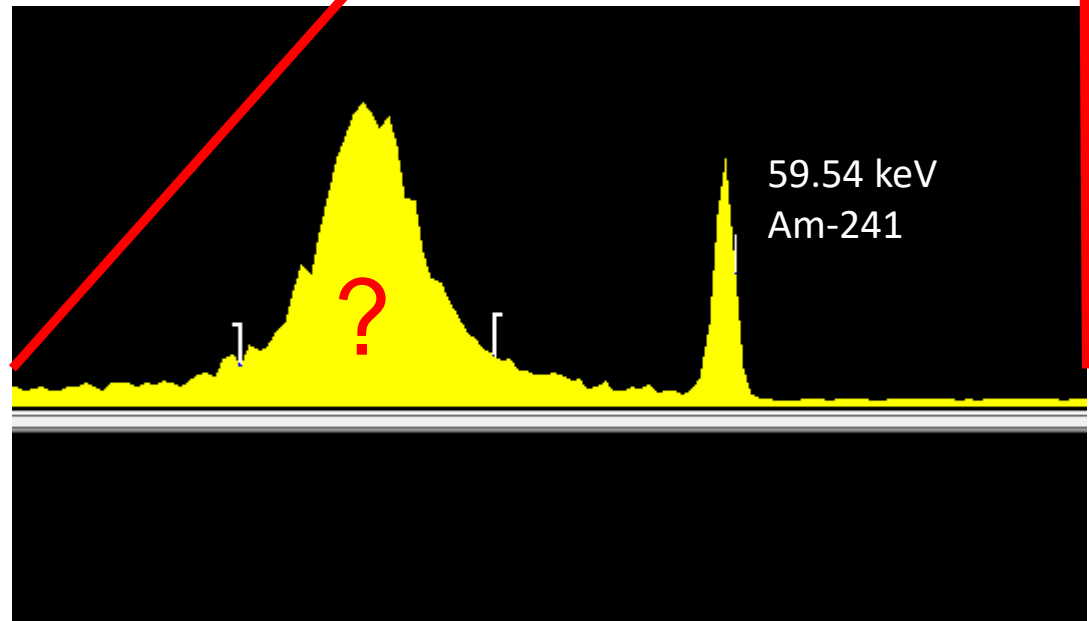
$$\mu_l = -\frac{\ln\left(\frac{R_0}{R}\right)}{x}$$

- High intensity sources required
- At low energy – small angle Compton scattering contributions may bias the results
- Time consuming

# Effects of low-angle scattering may complicate multi energy transmission



Spectrum obtained by using a single collimator





# Attenuation Correction Procedures based on Transmission

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- NUREG/CR-5550 (LA-UR-90-732) "Passive Nondestructive Assay of Nuclear Materials", 1991
  - Chapter 6: "Attenuation Correction Procedures" by J.L. Parker and references therein
  - **Far-field** form for self-attenuation correction factor for a box-shaped sample

$$CF(AT) = \frac{\mu_l x}{1 - \exp(-\mu_l x)}$$

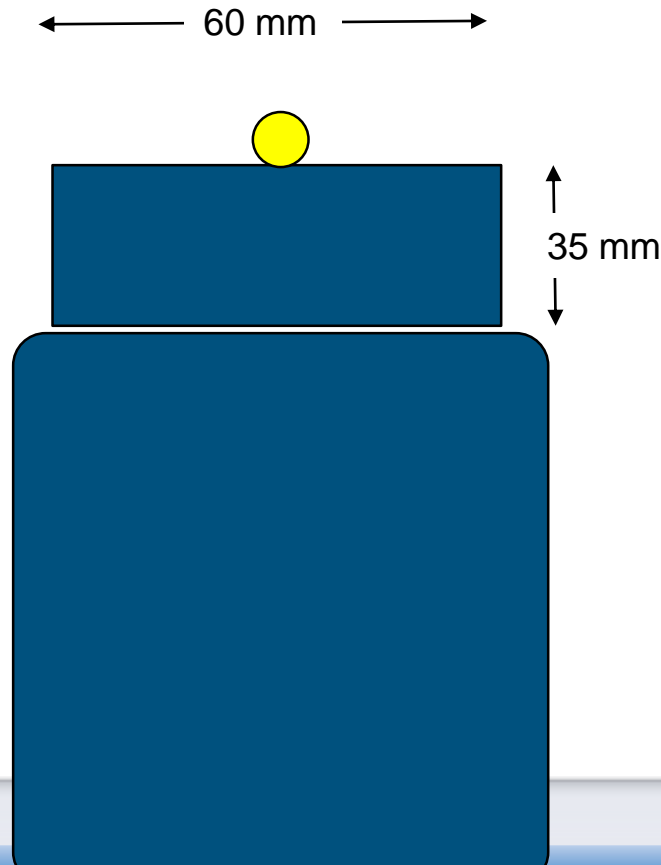
$$CF(AT) = \frac{-\ln(T)}{1 - T}$$

- These equations are only valid if the gamma-rays travel in a perpendicular direction from the sample towards the detector
- This is generally not the case, especially not when the sample is measured close to the detector

# Attenuation Correction Procedures

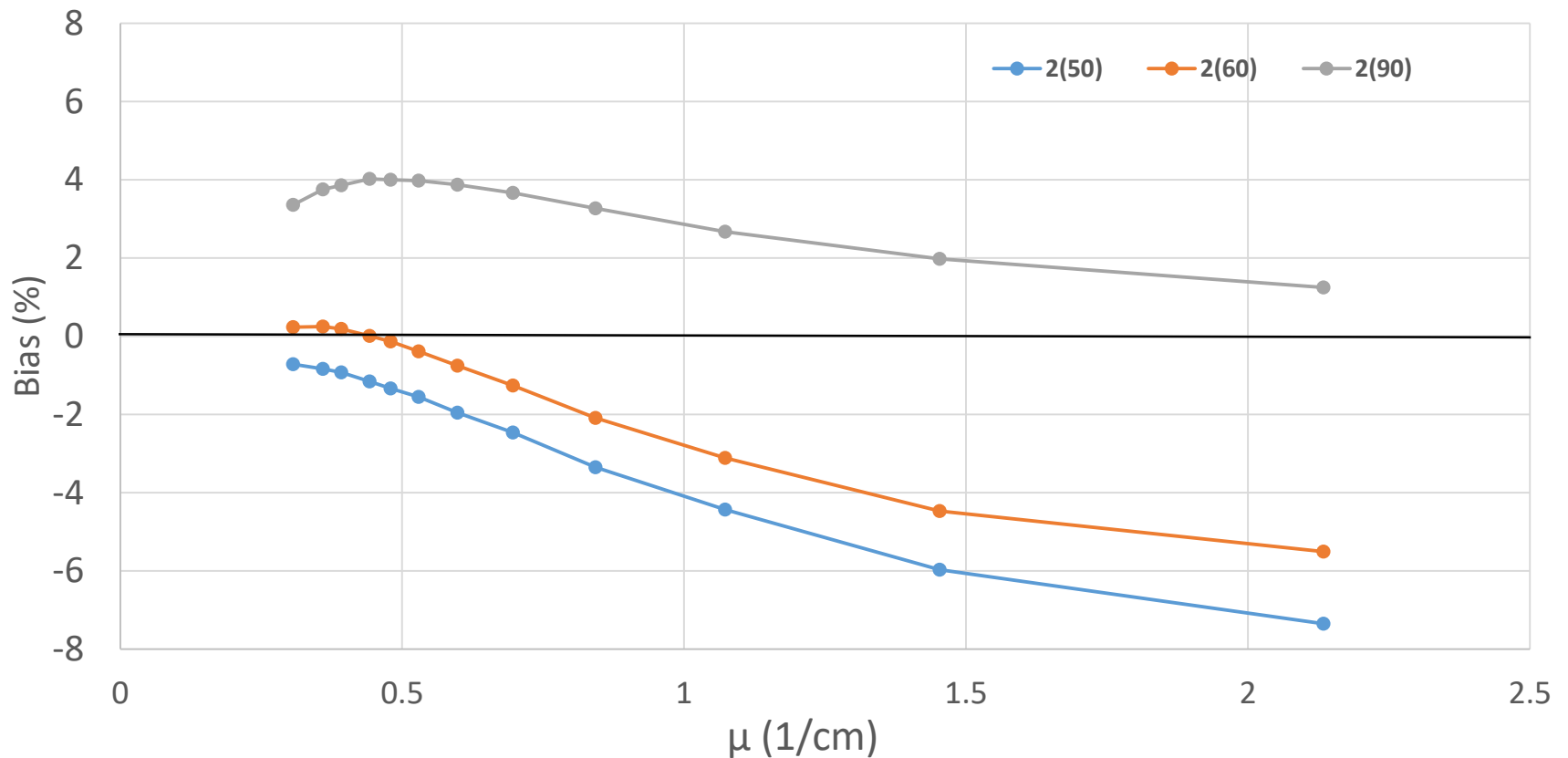
- N.H. Cutshall, correction for  $\text{Pb}^{210}$  in sediments (1981)
- **Near field (wide angle)**

$$CF(AT) = \frac{-\ln(T)}{1 - T}$$



# Cutshall correction (relative to air) compared to EFFTRAN

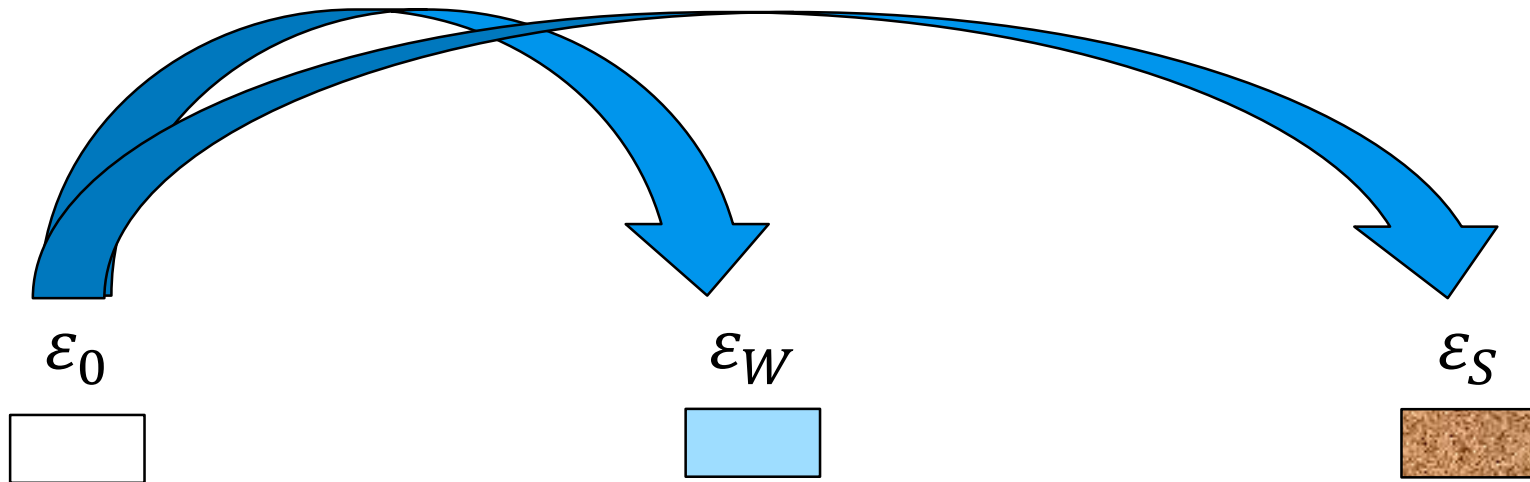
Bias cutshall vs. EFFTRAN (%) for different geometries



Sediment sample in pillbox (20mm height at different diameters, 50mm, 60mm and 90mm)  
Cutshall relative to air  
On a 60mm x 60mm detector

# Cutshall relative to another reference sample material

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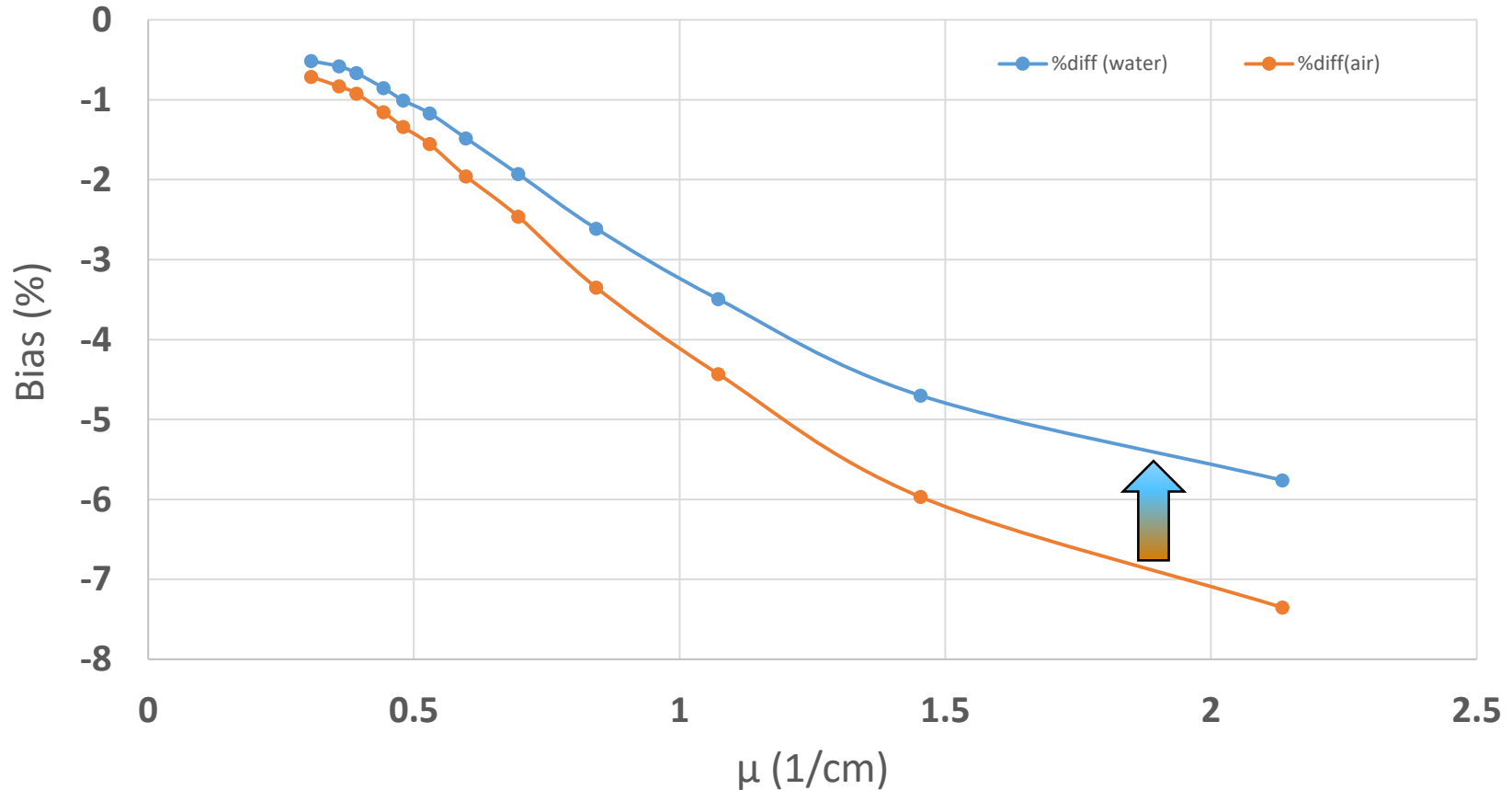
$$\epsilon_W = \frac{-\ln(T_W)}{(1 - T_W)} \epsilon_0$$

$$\epsilon_S = \frac{-\ln(T_S)}{(1 - T_S)} \epsilon_0$$

$$\epsilon_S = \frac{\ln(T_S)(1 - T_W)}{\ln(T_W)(1 - T_S)} \epsilon_W$$

# Transmission relative to reference closer to the sample improves accuracy

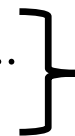
bias (%) as a function of  $\mu$  and choice of reference



# What to do to get unbiased results ?

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- Cutshall method may give acceptable results (few % bias)
  - Specially for not too high attenuation
  - Well selected geometry
  - Live with the bias and include it in the standard uncertainty
- Generally Cutshall is biased
  - Requires additional corrections
- Better to use full simulation to fix the relation between transmission data and sample material ( $\mu$ -values) for a specified geometry
  - General purpose Monte Carlo codes like MCNP,...
  - Gespecor
  - TEFTRAN (by Tim Vidmar)



Not easy to include  
in automation or  
simple working  
procedure

# Single energy versus energy range corrections

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## ● **Single Energy Correction**

- Correct the required energy e.g. 46.54 keV for Pb-210
- Use a transmission source that exactly emits this energy

## ● **Corrections in an Energy Range**

- Multi energy gamma-ray source
- Low energy peaks suffer bad counting statistics when measured with a multi-energy gamma-ray source
- Only the low energy window 30 keV – 120 keV requires element specific corrections, for the higher energy a correction based on density only is appropriate
- What is an appropriate multi energy source ?
  - I-129 – Am-241
- Interpolation between transmission/attenuation data
- How to deal with K,L...-edges

# Inter/extra-polation of attenuation data

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- 1) Via polynomial fitting of the attenuation data as a function of energy (= a strongly varying function of energy at low energy)
- 2) Via fitting with an appropriate material composition

$$\mu(E) \approx \sum_{i=1}^N w_i \mu_i(E)$$

With  $\mu_i(E)$  the mass attenuation coefficient for element  $i$  from XCOM

With  $w_i$  the relative contribution of element  $i$  and  $\sum w_i = 1$

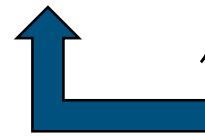
With  $N$  the number of elements in a well defined set of elements



# Inter/extra-polation of attenuation data

- General least squares fitting minimizing F by changing  $w_i$

$$F(E_j, w_i) = \left( \mu_{exp}(E_j) - \sum_{i=1}^N w_i \mu_i(E_j) \right)^2 = 0$$



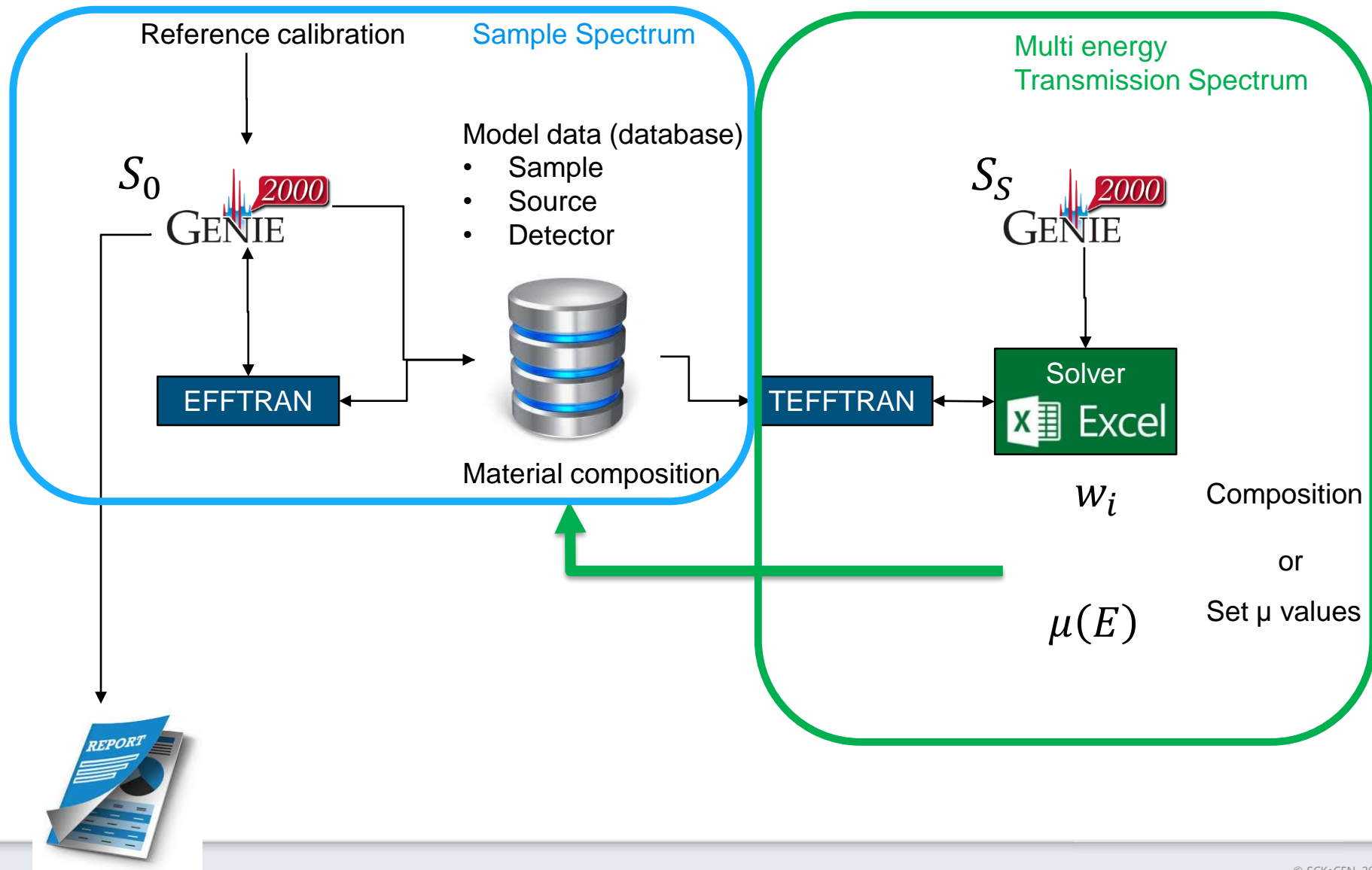
- The set of elements should be well selected
  - Not complete periodic table
  - Element sets can be defined for specific materials (weak point)
- Least squares problem can be solved in e.g. Excel (Solver add-in)
- Outcome is not a polynomial but a sample composition
  - Inter & extrapolation (by XCOM)
  - K-edges are included (if elements data set is correct)
- The computed sample composition is not necessary the correct composition but yields an equivalent composition

# TEFFTRAN: the way to $\mu$ -values from uncollimated experiments

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- Uses computational procedures like in EFFTRAN
- Considers a cylindrical source (dimensions)
- Source is on top of sample (cutshall geometry)
- Sample is a cylinder (all walls same thickness, composition and density are to specify)
- Detectormodel as in EFFTRAN
- No absolute response but a ratio of two computations at different density and composition is to be compared with the experimental ratio
- TEFFTRAN can be run from a batch command once the input files are defined
  - In an iteration process the  $\mu$ -value can be obtained from the simple transmission measurement

# Procedure for low-energy attenuation correction based on TEFFTRAN and EFFTRAN



# Combining transmission data and element composition

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- To fill the gap of missing elements in the element composition (H-Na, Mg-S)
- Use the measured heavy elements to compose a basic composition
  - Refine the composition with missing elements, H, O, C,...
  - Compare the measured  $\mu$ -data with the computed  $\mu$ -data
- Make efficiency transfer with the optimized composition
- Uncertainty component  $\Delta\mu \rightarrow \Delta CF$

- Gamma-ray attenuation data can be obtained from a transmission experiment without collimator using an appropriate modeling software of TEFFTRAN
- Multi Energy Transmission data only do not give full details on actual gamma-attenuation
  - No means to account for K-edges
  - Determination of an equivalent composition with an appropriate set of elements may (partially) cope with K-edges
  - A combination of transmission + the measurement of the element composition of the sample material (X-ray,...) may result in the most complete information
- A procedure for efficiency transfer relaying on EFFTRAN and TEFFTRAN was proposed

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Thank you for your attention