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Uncertainties

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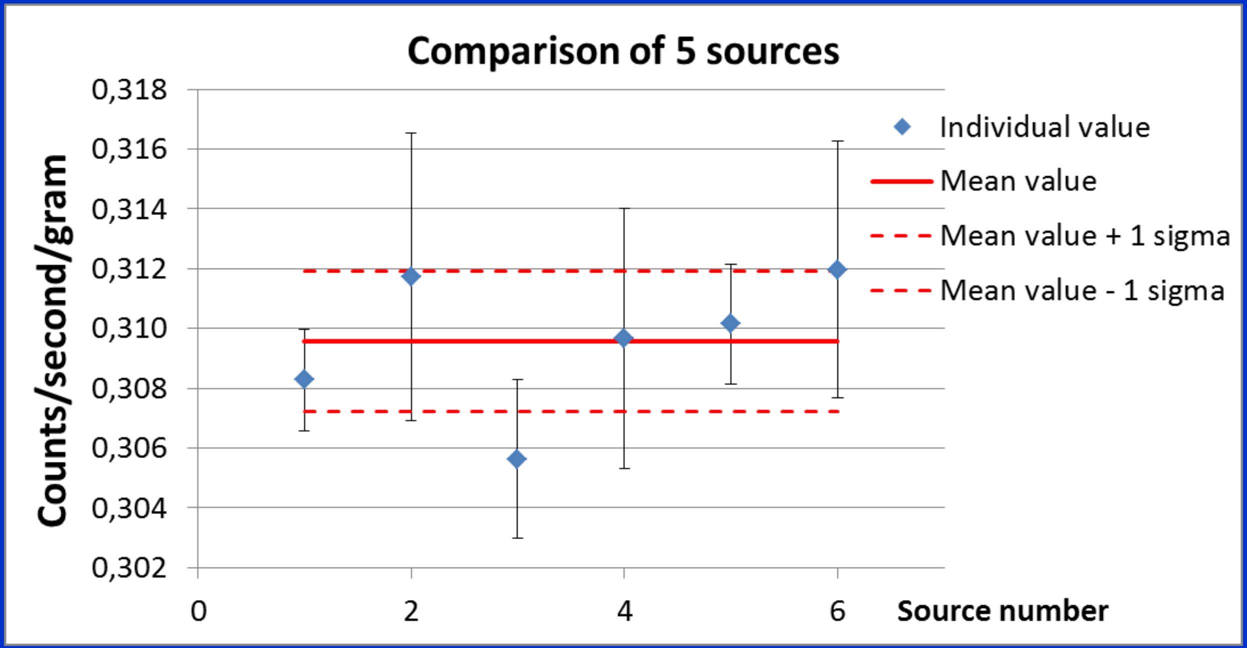
Uncertainties

- The quality of a result obtained by gamma-ray spectrometry (efficiency, activity) depends on:
  - Source (sample) preparation
  - Measurement setup (electronics)
  - Spectrum processing
  - Corrective factors
- Associated uncertainties should reflect all these aspects

Sample representativity

Example: point source from a standard solution to determine efficiency

5 point sources (^{109}Cd) with masses m1, m2, m3, m4, m5



Measuring each source -> count rate per second and per gram

Sample representativity

What about environment samples ? :

What do I wish to measure ?

Is my sample representative of that ?

-Sampling
- Preparation
- Position – Filling height
- Container bottom thickness
    - change the source-to-detector distance
    - attenuation change
- Homogeneity

Electronics

DSP (Digital Signal Processing modules)
Automatic settings
Lot of parameters

^{60}Co + ^{137}Cs

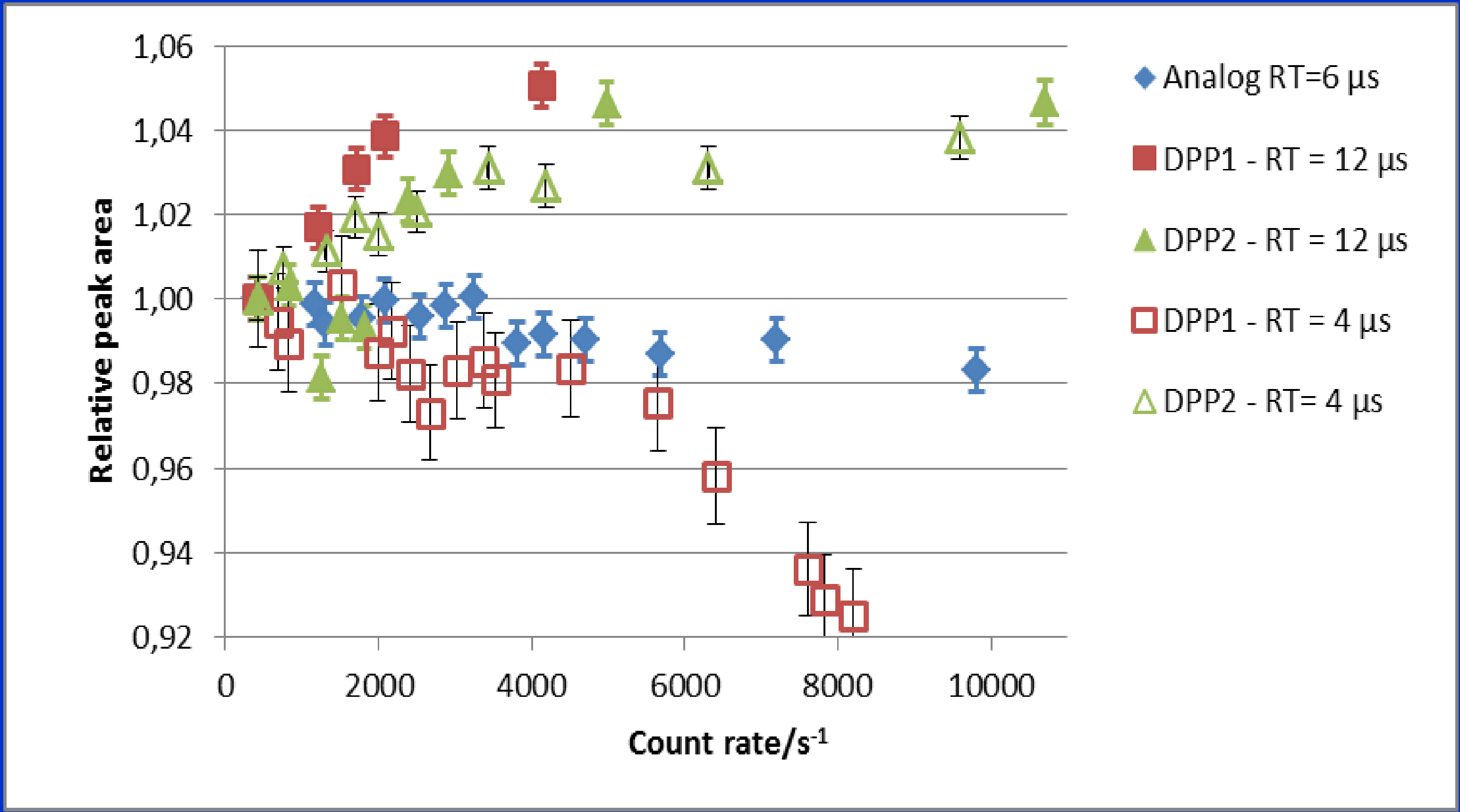
^{133} Ba

^{60}Co and ^{137}Cs: reference:
fixed position

^{133}Ba: moved close to detector window to increase the count rate

Tests of electronics

Evolution of the relative 1332 keV peak area versus the count rate for different electronics



Associated uncertainties

- $\varepsilon_i = \frac{N_i \cdot \prod C_{ij}}{A \cdot I_i}$

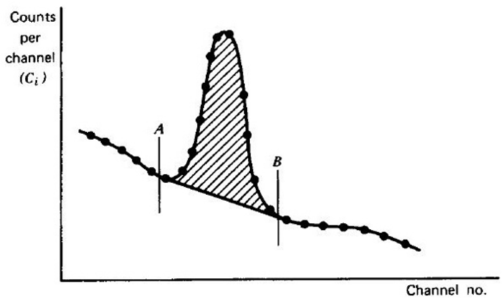
$$\frac{u(\varepsilon_i)}{\varepsilon_i} = \left[\frac{u^2(N_i)}{N_i^2} + \frac{u^2(A)}{A^2} + \frac{u^2(I_i)}{I_i^2} + \sum_j \frac{u^2(C_{ij})}{C_{ij}^2}\right]^{1/2}$$

Example: Point source of ^{137}Cs - Efficiency at 661.7 keV
u(A) / A = 0.5 %.
u(Ii) / Ii : 0.24 %.
In the best experimental conditions, where there are no corrective factors, short acquisition time (in comparison with the ^{137}Cs half-life).

Relative uncertainty on peak area (%)	Relative uncertainty on FEP efficiency (%)
1	1.14
0.1	0.56

If u(A) / A = 0.2 %, the FEP efficiency can be obtained with 0.3 % relative uncertainty. This is the minimum that can be experimentally achieved in this very favorable case.

Peak area uncertainty



$$\frac{u^2(N_i)}{N_i^2} = \frac{u^2(A_i)}{A_i^2} + u^2(C_A)$$

Depending on
' the background shape
the user (defintion of binding channels
…

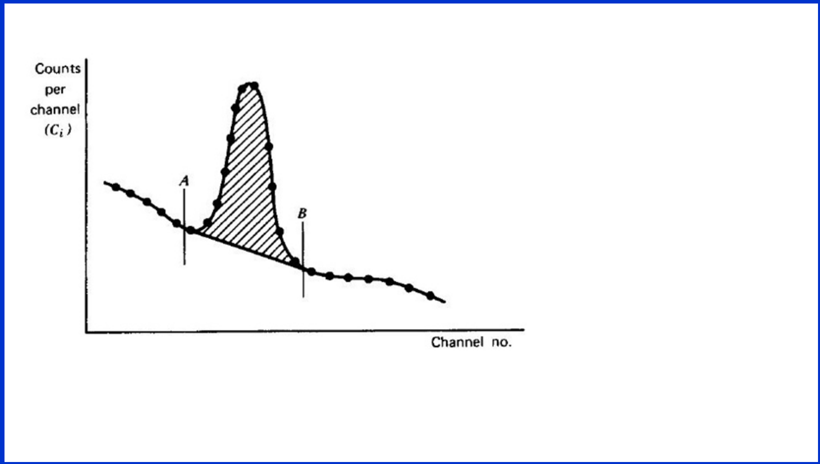
Can be considered as the sum of :

True value of the peak area Ai, and associated uncertainty (statistics) $A \pm u(A)$
Corrective factor , C_A (=1 if no better data)

The corrective factor generally cancels out when the same procedure is used in calibration and measurement

Peak area determination

- Summing method



- Net = Sum – Background
- $N = N_S - N_B$
- $u(N) = \sqrt{N_S + N_B}$

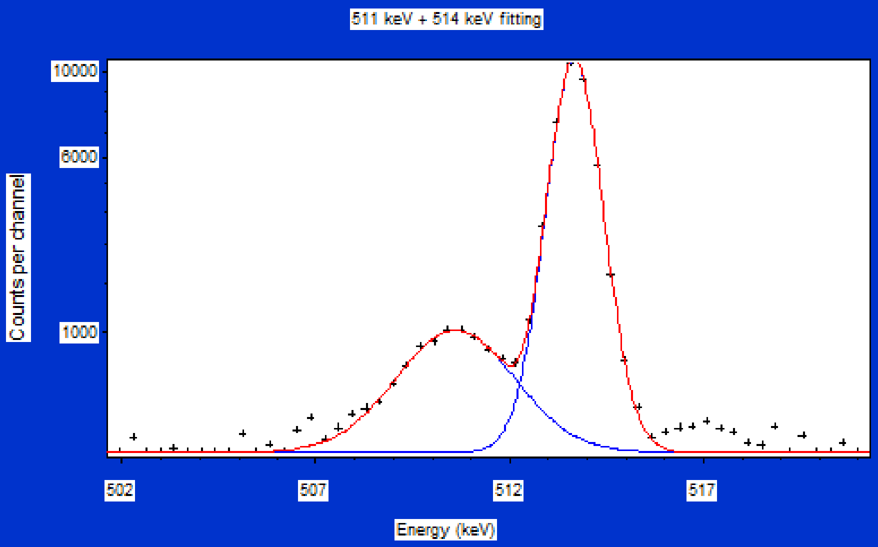
- Different background shapes

Peak area determination

Fitting method

Using a mathematical method (least squares) to fit a Gaussian to the peak

$$G(E) = G \cdot \exp\left(-\frac{(E - E_O)^2}{2\sigma^2}\right)$$



The total Gaussian area is

$$S(E) = \sqrt{2\pi}\,\sigma \cdot G$$

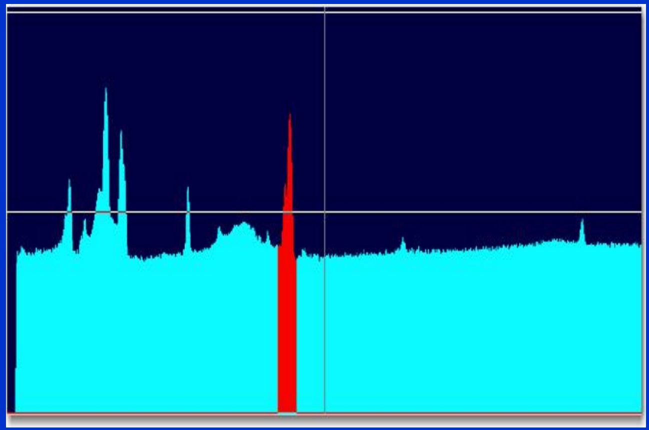
with the associated uncertainty:

$$\frac{u(S(E))}{S(E)} = \left[\frac{u^2(\sigma)}{\sigma^2} + \frac{u^2(G)}{G^2}\right]^{1/2}$$

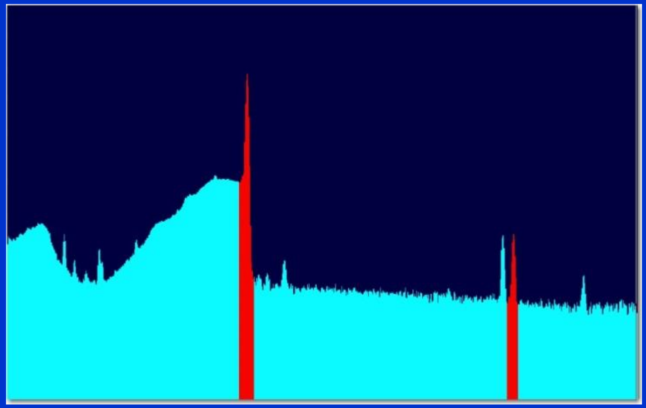
S(E) is the Gaussian area integrated over the energy range [-∞, +∞]. However, 99 % of the area is within the [E_0 -2.58 σ, E_0 + 2.58 σ] interval, what can be considered as the practical peak area.

Peak area determination

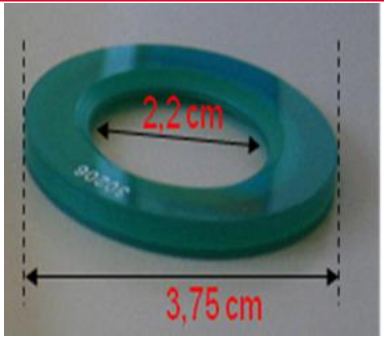
- Scattering effect (low energy range)
- Volume sources



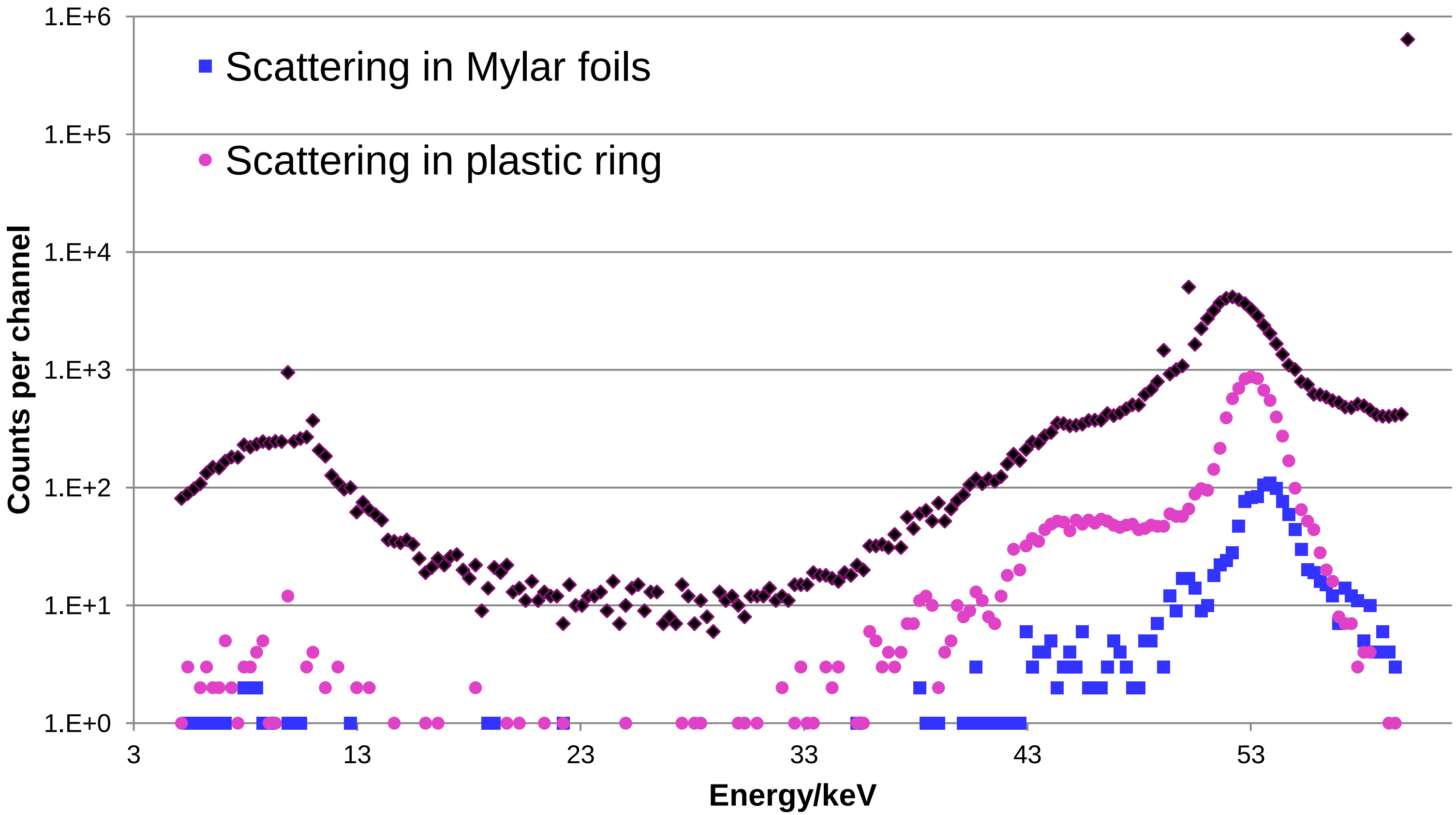
- Example :
  - ^{133}Ba (point source)
  - ^{133}Xe (gaz)
  - Same line at 80 keV



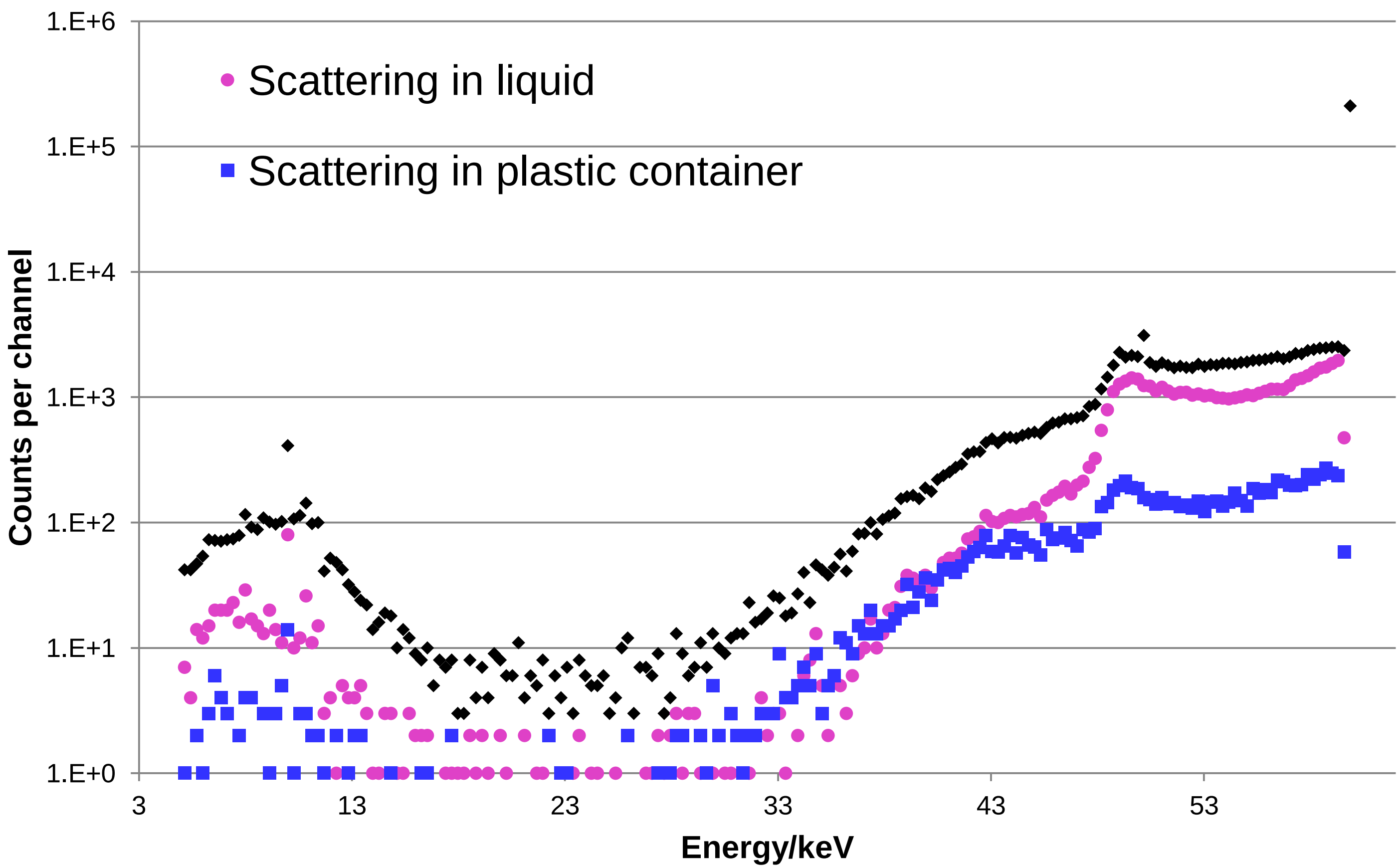
- Efficiency transfer questionable



Monte Carlo simulation for 60 keV photons
Point source at 10 cm



Monte Carlo simulation for 60 keV photons:
Solution (H_2O) in a 50 cm^3 plastic container at 10 cm



Corrective factors

- Half-life – decay during measurement
- Attenuation
- Self-attenuation
- Geometry
- Coincidence summing
- Background
- Escape peaks
- Dead-time
- Annihilation in-flight (beta +)

Corrective factors

- Half-life

$$C_T = exp\left(-\ln(2)\cdot\frac{(T_m - T_r)}{T_{1/2}}\right)$$

- T_m is the measurement time (when the measurement is carried out),
- T_r is the reference time (when the reference activity is known),
- $T_{1/2}$ is the radionuclide half-life.
- It is assumed that the nuclide is not a member of a decay series.

$$\frac{u(C_T)}{C_T} = \ln(2)\cdot\frac{(T_m - T_r)}{T_{1/2}}\cdot\frac{u(T_{1/2})}{T_{1/2}}$$

The uncertainty on the measurement/reference time is generally negligible

Corrective factors

- Decay during measurement (Short half-life)

$$C_{Dec} = \frac{\ln(2) \cdot \frac{t_r}{T_{1/2}}}{1 - exp\left(-\ln(2) \cdot \frac{t_r}{T_{1/2}}\right)}$$

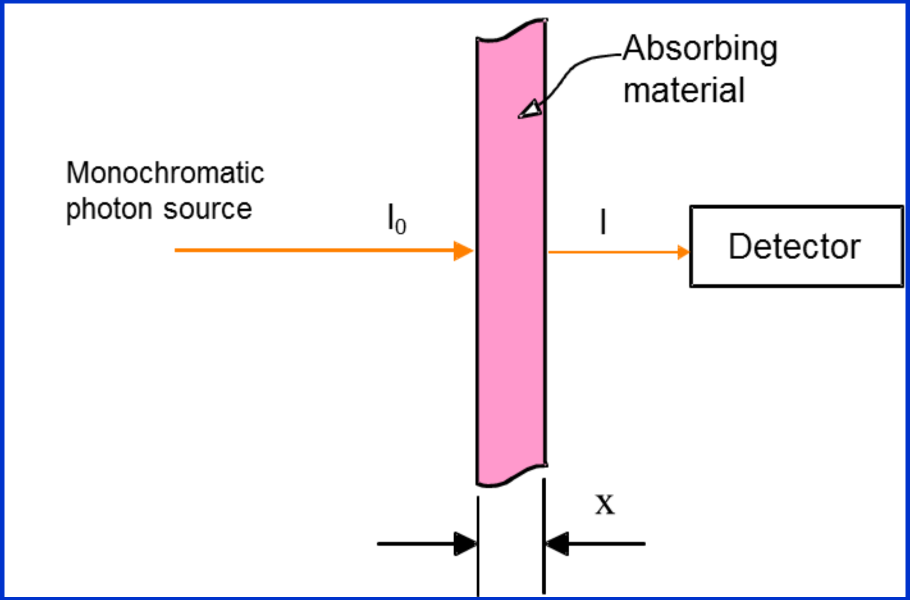
t_r is the real acquisition time.

$$\frac{u(C_{Dec})}{C_{Dec}} = \left[1 + C_{Dec} \cdot exp\left(-\ln(2) \cdot \frac{t_r}{T_{1/2}}\right)\right] \cdot \frac{u(T_{1/2})}{T_{1/2}}$$

Geometry corrections

- Difference betwwen the calibration and measurement conditions
  - Attenuation (screen)
  - Geometry (distance, filling height)
  - Self-attenuation (matrix, filling height)

Attenuation



$$I(E) = I_0(E) \cdot exp(-\mu(E) \cdot x)$$

$\mu(E)$ and $\mu/\rho\ (E)$ are respectively the linear and the mass attenuation coefficients of the screen material and ρ is its density.

$$C_{Att}(E) = exp(-\mu(E) \cdot x) = exp\left(-\frac{\mu}{\rho}(E) \cdot \rho \cdot x\right)$$

$$\frac{u(C_{Att}(E))}{C_{Att}(E)} = \mu(E) \cdot x \sqrt{\left(\frac{u(x)}{x}\right)^2 + \left(\frac{u(\mu(E))}{\mu(E)}\right)^2}$$

Valid only for monochromatic photons arriving under normal incidence on the absorbing layer.

Self-attenuation

- Integrating the Beer-Lambert law:
- Simplified correction

$$C_{Self}(E) = \frac{1 - exp(-\mu(E) \cdot x)}{\mu(E) \cdot x}$$

$$\frac{u(C_{Self}(E))}{C_{Self}(E)} = \left| \frac{\exp(-\mu(E) \cdot x)}{C_{Self}(E)} - 1 \right| \cdot \sqrt{\left(\frac{u(x)}{x}\right)^2 + \left(\frac{u(\mu(E))}{\mu(E)}\right)^2}$$

Approximation valid for small volume and large source-to-detector distance

Practical tools for geometry corrections

- Change of geometry -> change of efficiency
- Efficiency transfer corrections

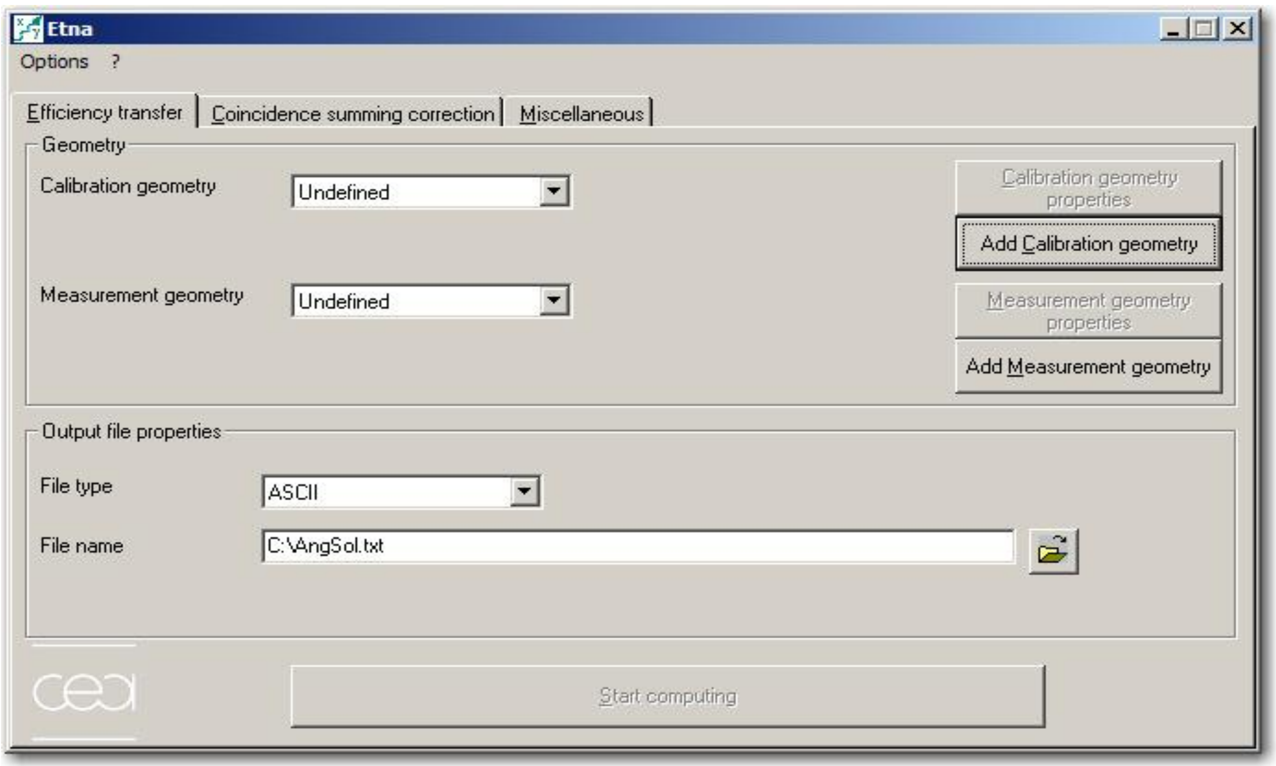
- Pure Monte Carlo methods
- GESPECOR, LabSOCS™ (commercial)
- Numerical methods (Moen’s principle) dedicated software such as EFFTRAN, ANGLE (commercial) or ETNA

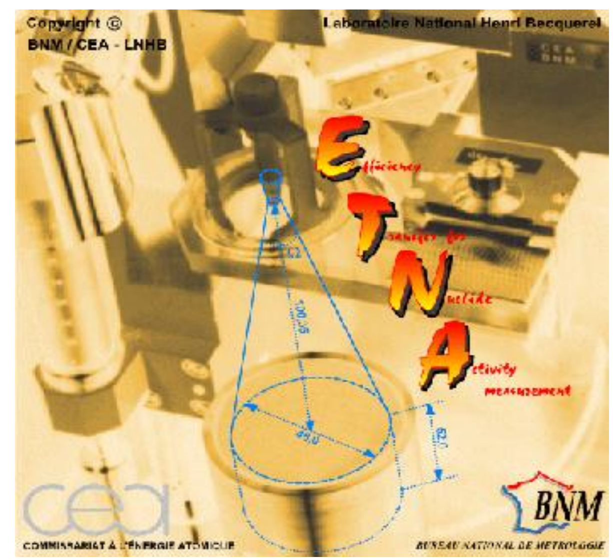
ETNA

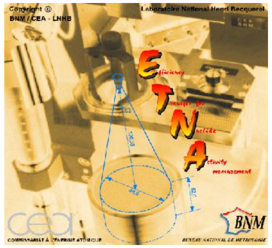
(Efficiency Transfer for Nuclide Activity measurement)

ETNA is a software for computing efficiency transfer and coincidence summing corrections for gamma-ray spectrometry.

The software has been developed at the Laboratoire National Henri Becquerel and is available upon request.



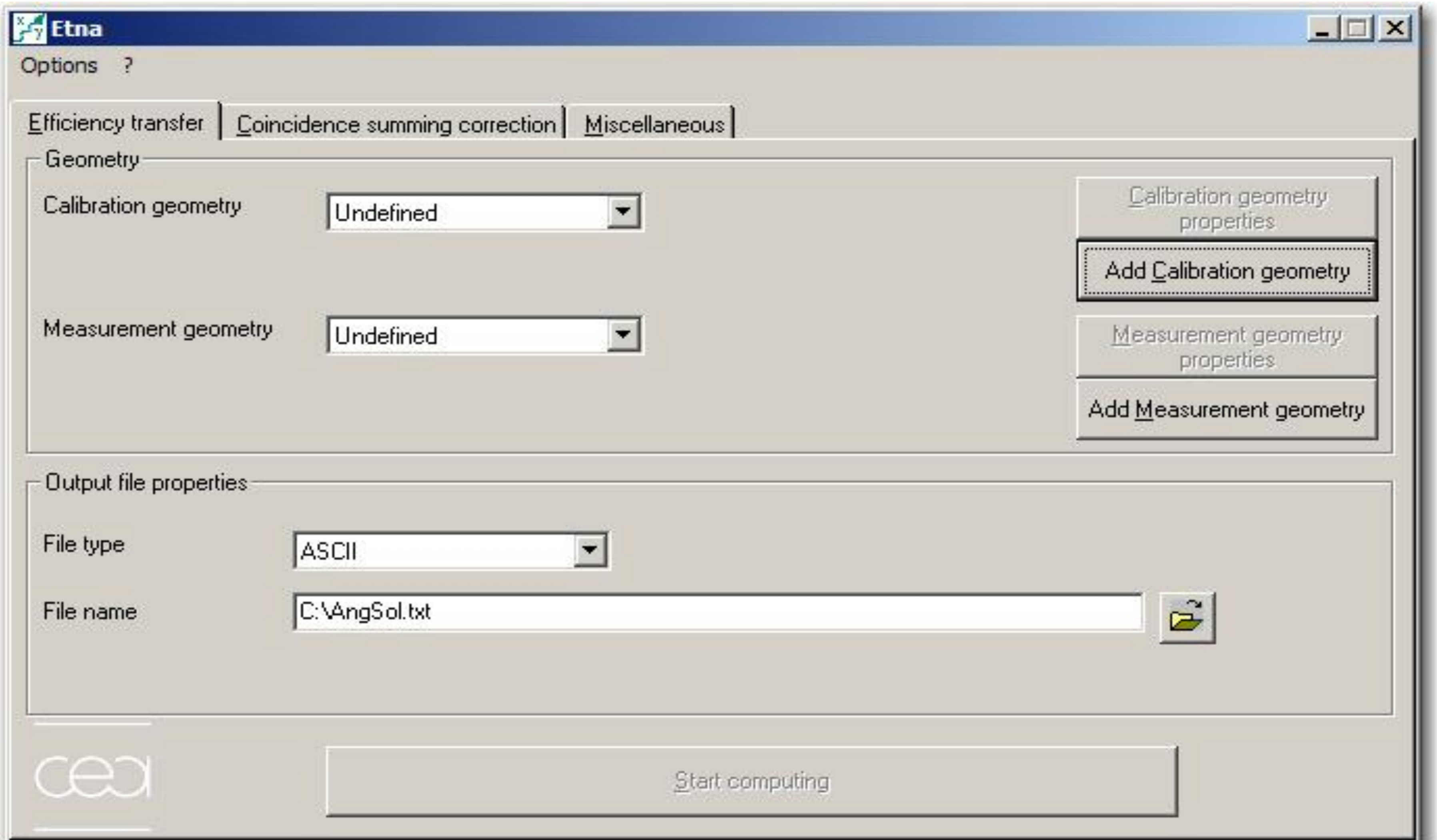




ETNA

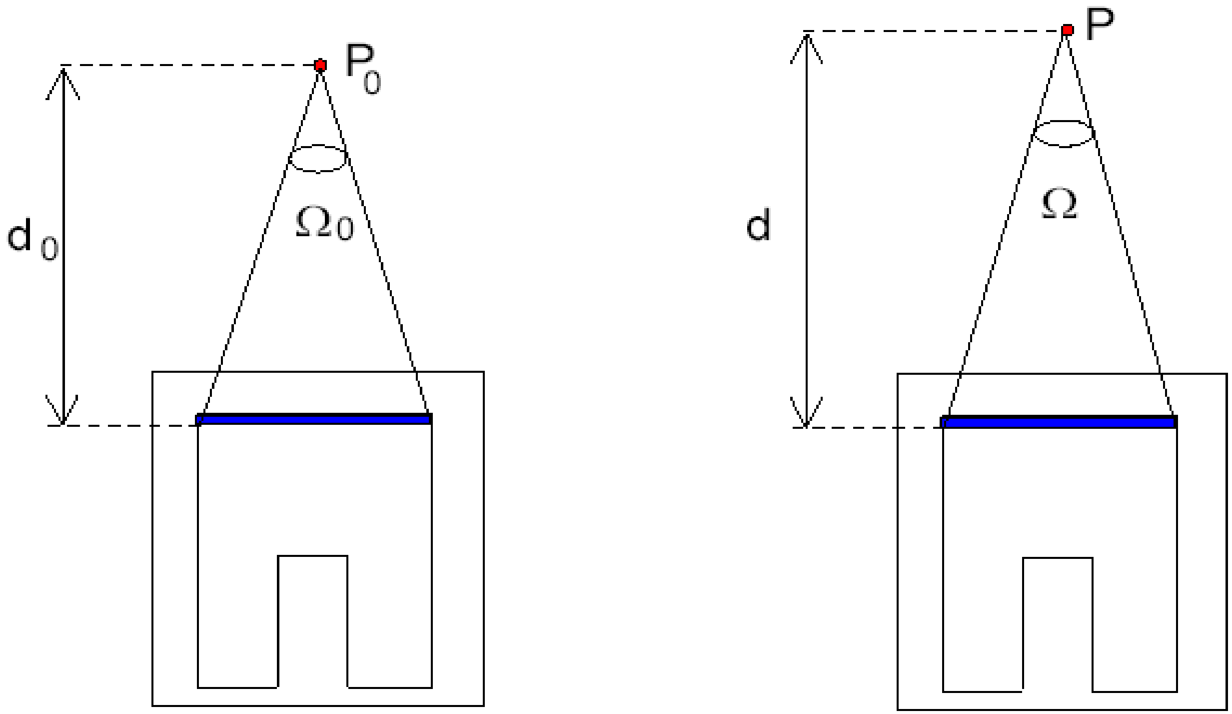
- Transfer of efficiency
  - Semi-empirical method (from a reference efficiency)
  - Coaxial cylindrical geometry (point. disk. cylinder. Marinelli)
- Coincidence summing corrections
  - Knowledge of the efficiency (total and full-energy peak)
  - Possibility of efficiency transfer
  - Decay scheme from Nucleide
- Data management
  - Decay scheme
  - Attenuation coefficients

ETNA main window



Efficiency transfer principle

Point source moving along the detector axis



$$\mathcal{E}(E,P_0)=\mathcal{E}I(E).\Omega(P_0) \qquad \mathcal{E}(E,P)=\mathcal{E}I(E).\Omega(P)$$

$$\mathcal{E}(E,P)=\mathcal{E}(E,P_0).\frac{\Omega(P)}{\Omega(P_0)}$$

Solid angle for point source

Using polar coodinates. the solid angle Ω(P) between point P (r, ϕ, z_s) and the detector entrance surface (disc) is:

$$\Omega(P) = 2 \cdot z_S \int_0^{\pi} d\varphi \int_0^{R_D} \frac{R \cdot dR}{\left[R^2 - 2 \cdot R \cdot r \cdot \cos\varphi + r^2 + z_S^2\right]^{3/2}}$$

R_D is the detector radius.

The geometrical factor should include:

- attenuation in differents absorbing layers (air, window, dead layer. …) : F_{att}

$$F_{att} = \exp\left(-\sum_{i=1}^{m} \mu_i \cdot \delta_i\right)$$

- absorption in the detector active volume : F_{abs}

$$F_{abs} = f_1 + f_2 \cdot f'$$

$$f_1 = 1 - \exp\left(-\mu_D \cdot \delta_{1D}\right) \qquad f_2 = 1 - \exp\left(-\mu_D \cdot \delta_{2D}\right) \qquad f' = \exp\left(-\mu_D \cdot (\Delta + \delta_{1D})\right)$$

Solid angle for a cylindrical source

- For a volume source (cylindrical symmetry : radius $R_{S,}$ thickness $H_{S,}$ vertical position Z_S):

$$\Omega = \frac{4}{R_S^2 \cdot H_S} \int_{Z_S}^{Z_S+H_S} h \cdot dh \int_0^{R_S} r \cdot dr \int_0^{\pi} d\varphi \int_0^{R_D} \frac{R \cdot dR}{\left[R^2 - 2 \cdot R \cdot r \cdot \cos\varphi + r^2 + h^2\right]^{3/2}}$$

- Fatt and F abs must be included in the integration procedure

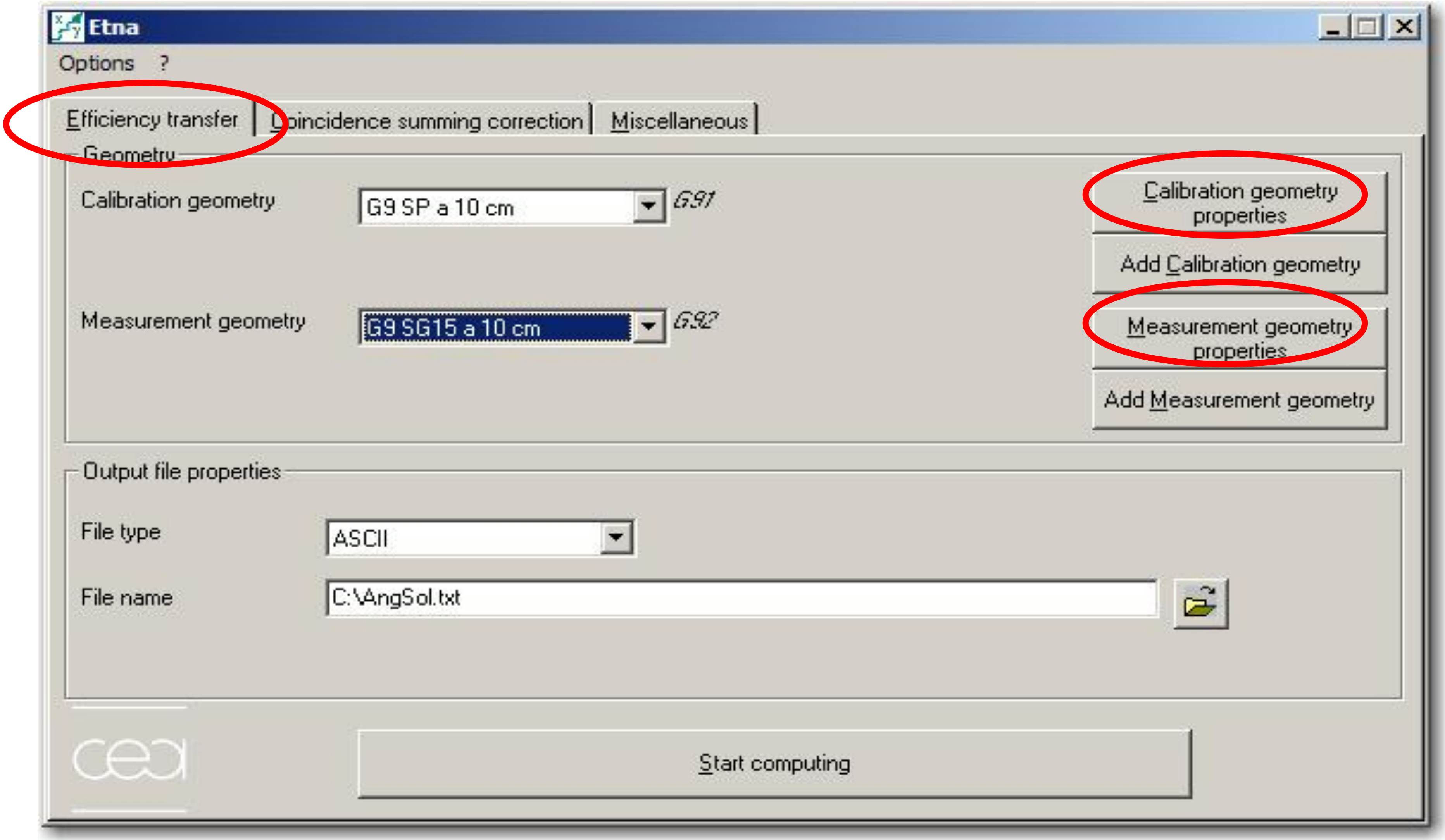
Integration are numerically performed using the Gauss-Legendre method.

Point sources, discs, cylinders and Marinelli (along the detector axis) are considered.

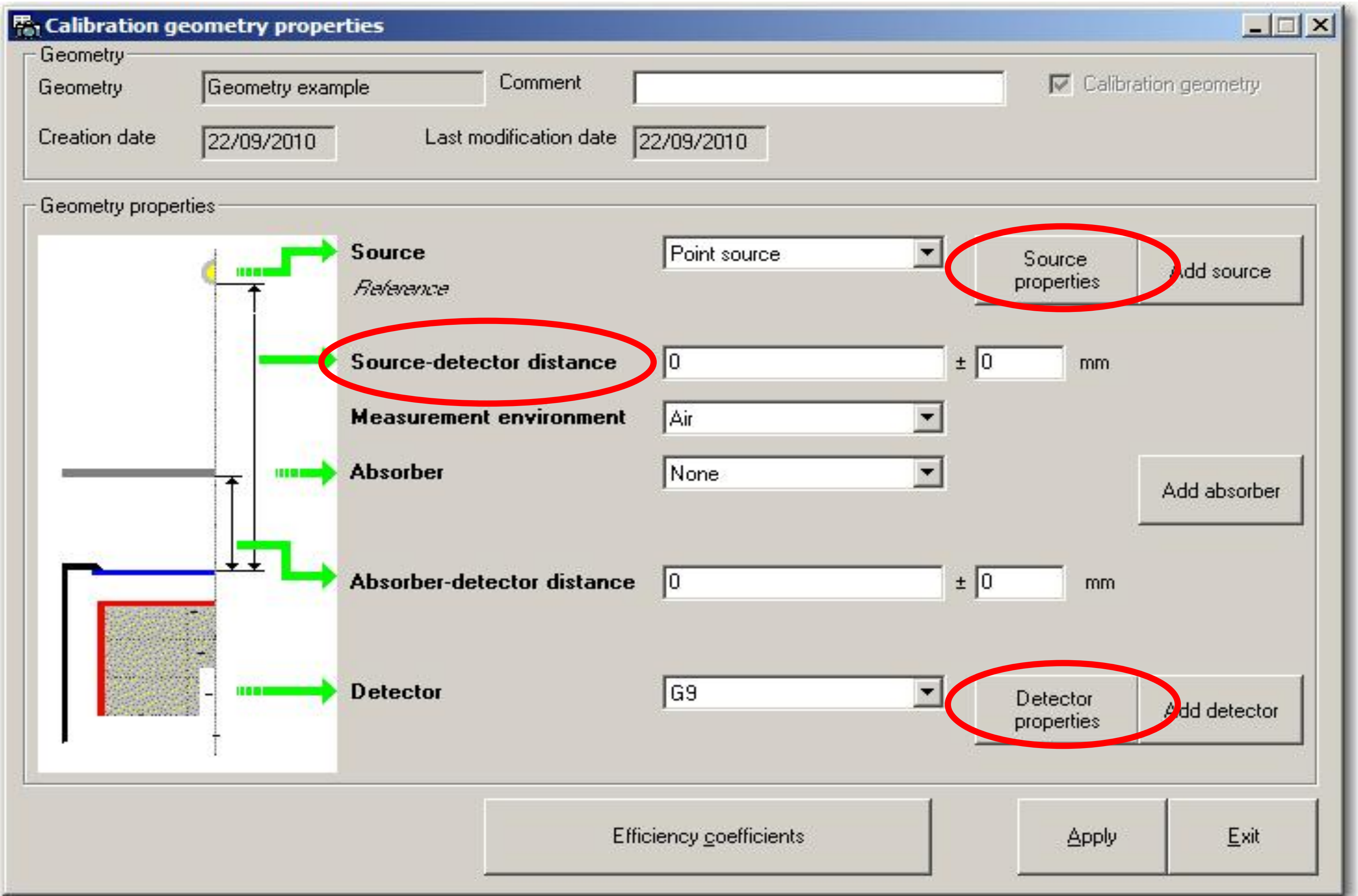
Input of data

- Requires
  - Detector parameters
  - Source parameters
    - Container
    - Matrix
  - Geometry conditions (source-to-detector distance, screen)
  - Reference efficiency
- Recorded in the « user » database

Efficiency transfer window



Input of geometry parameters



Efficiency transfer results

ETNA Version 5.5 Rev 55

Filename : C:\Documents and Settings\LEPY.BECQUEREL\Bureau\test

jeudi 23 septembre 2010
Processing identification : Efficiency transfer
Calibration geometry : G9 SP at 10 cm (G91)
Calibration source : POint source (Reference)
Calibration source - detector distance : 103.4 mm
Calibration absorber : None
Calibration absorber - detector distance : 0 mm
Measurement geometry : G9 SG15 a 10 cm (G92)
Measurement source : Volume SG15 (Filled with HCl 1N 41 mm)
Measurement source - detector distance : 104 mm
Measurement absorber : None
Measurement absorber - detector distance : 0 mm
Detector : G9

Results :

Energy (keV)	Calibration efficiency	Measurement efficiency	Ratio
00020.000	00.00288000	00.00058685	00.20376736
00050.000	00.00812000	00.00391170	00.48173645
00080.000	00.00932000	00.00482243	00.51742811
00100.000	00.00924000	00.00489268	00.52951082
00120.000	00.00882000	00.00476234	00.53994785
00150.000	00.00796000	00.00438473	00.55084548
00200.000	00.00655000	00.00371224	00.56675420
00250.000	00.00543000	00.00313296	00.57697238
00300.000	00.00458000	00.00268981	00.58729476
00400.000	00.00346000	00.00208513	00.60263873
00500.000	00.00278000	00.00170495	00.61329137
00750.000	00.00191000	00.00120835	00.63264398
01000.000	00.00149000	00.00096245	00.64593960
01250.000	00.00123000	00.00080666	00.65582114
01500.000	00.00105000	00.00069649	00.66332381
01750.000	00.00089500	00.00059884	00.66909497
02000.000	00.00076600	00.00051653	00.67432115

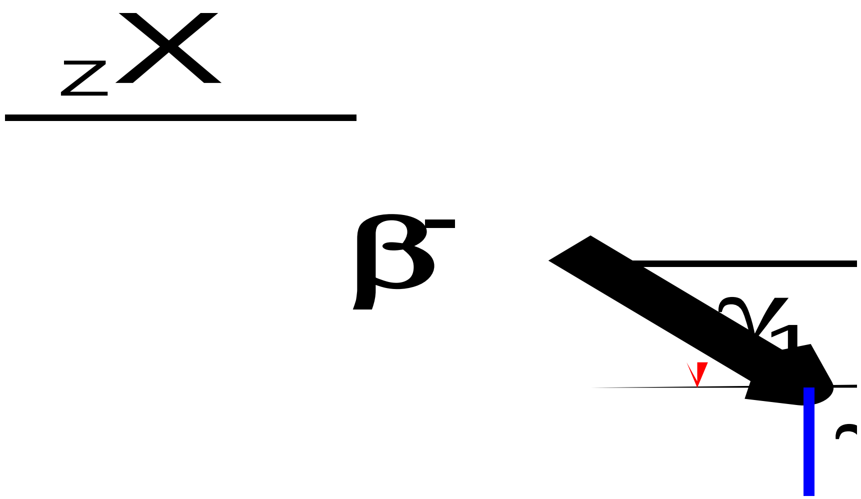
CEA / LNE-LNHB

Coincidence summing

- Effect due to the decay scheme
- Even at low counting rate
- More important at short source-to-detector distance
- Same kind of tools as for Efficiency transfer
  - Monte Carlo
  - GESPECOR
  - Dedicated numerical (ETNA, other ?)

Calculation principle

- ETNA uses a numerical method, according to Andreev, Mc Callum principle:



$$C_1 = \frac{1}{1 - P_{12} \cdot \eta_2}$$

$$C_2 = \frac{1}{1 - P_{21} \cdot \eta_1}$$

$$C_3 = \frac{1}{\left(1 + \frac{I_{\gamma 1}}{I_{\gamma 3}} \cdot \frac{\varepsilon_1 \cdot \varepsilon_2}{\varepsilon_3} \cdot P_{12}\right)}$$

P_{12} : probability for emitting γ_2 simultaneaously with γ_1

ε_i : FEP efficiency for energy E_i

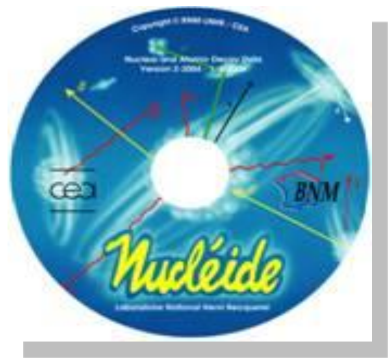
η_i : Total efficiency for energy E_i

Calculation principle (2)

- Double coincidences
- Coincidences with K X-rays (electron capture or internal conversion) are computed
- Correction for K-X-rays (from gamma or X rays) are computed
- Beta+ emitting nuclides are considered (modification of the decay scheme)
- No angular correlation

ETNA – Input data

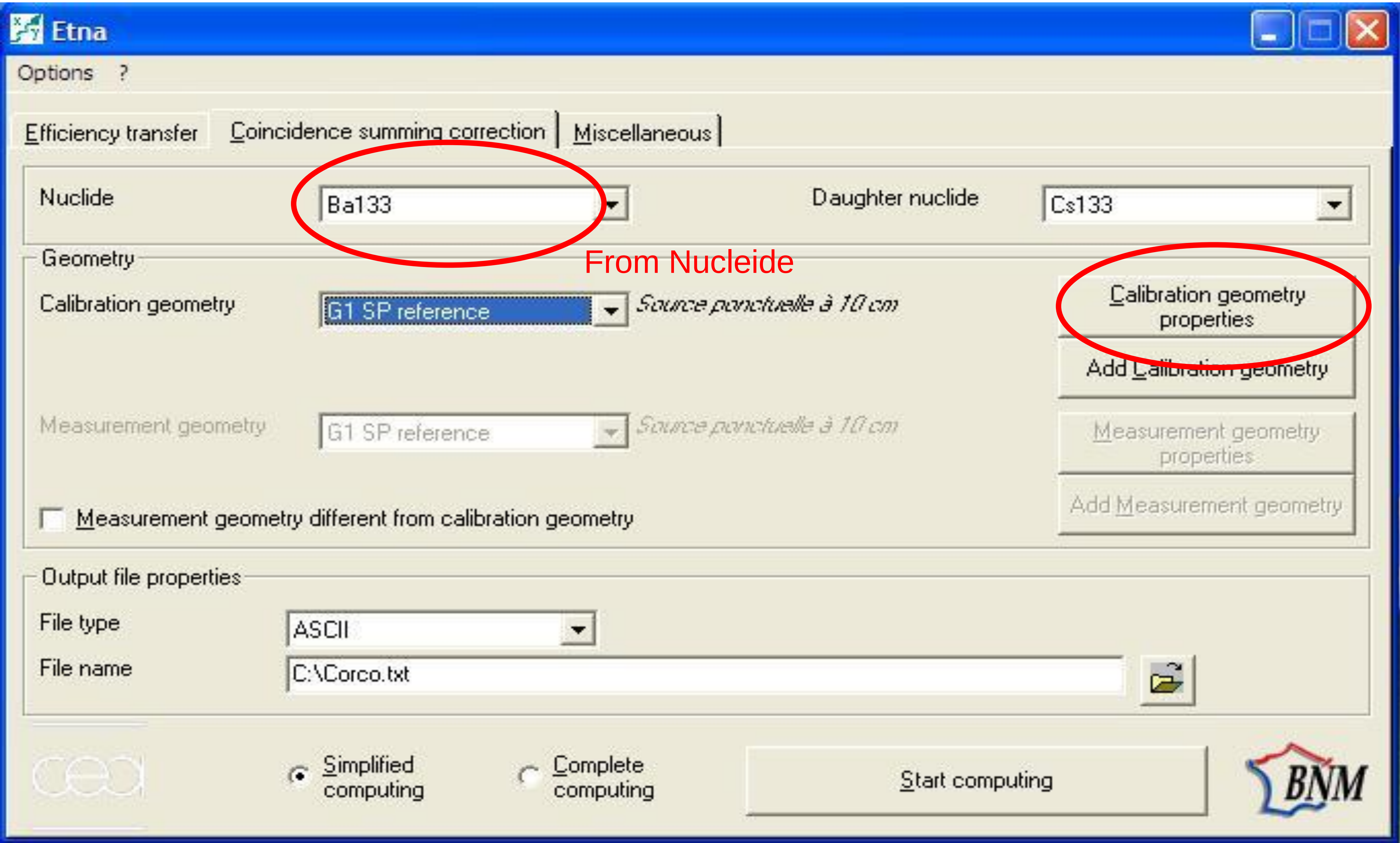
ETNA requires:



1. Decay scheme (Nucleide database)

2. FEP and total efficiency for at least one source-to-detector geometry («calibration geometry » recorded in the « user » database)

ETNA – Coincidence tab



Coincidence correction results

- dimanche 22 février 2009
- ETNA ____________________________________Version 5.5 Rev 51
- Filename :C:\Documents and Settings\ML118236\Bureau\Workshop_ICRM\Presentations\ETNA\test_ETNA
- dimanche 22 février 2009
- Processing identification : Coincidence summing correction (simplified computing)
- Nuclide :Ba133
- Daughter nuclide :Cs133
- Half-life threshold :0.000001 s
- Calibration geometry : G1 SP reference (Source ponctuelle à 10 cm)
- Calibration source :Source ponctuelle
- Calibration source - detector distance :100 mm
- Calibration absorber :None
- Calibration absorber - detector distance :0 mm
- Measurement geometry :Calibration geometry
- Detector :G1 - pièce 6A
- Results :
- Error codes : 0 0
- X-ray correction : 01.015880

Starting level	Arrival level	Energy (keV)	Gamma-gamma correction	Gamma-X correction	Total correction
004	003	00053.162	01.013962	01.010219	01.024324
002	001	00079.614	01.015207	01.012325	01.027720
001	000	00080.998	01.011478	01.007984	01.019554
002	000	00160.612	00.993490	01.007235	01.000678
003	002	00223.237	01.009461	01.019791	01.029439
004	002	00276.399	01.008560	01.015827	01.024522
003	001	00302.851	01.005028	01.015414	01.020519
004	001	00356.013	01.003565	01.011468	01.015074
003	000	00383.849	00.991597	01.010308	01.001818

- :
- CEA\LNHB ____ BNM

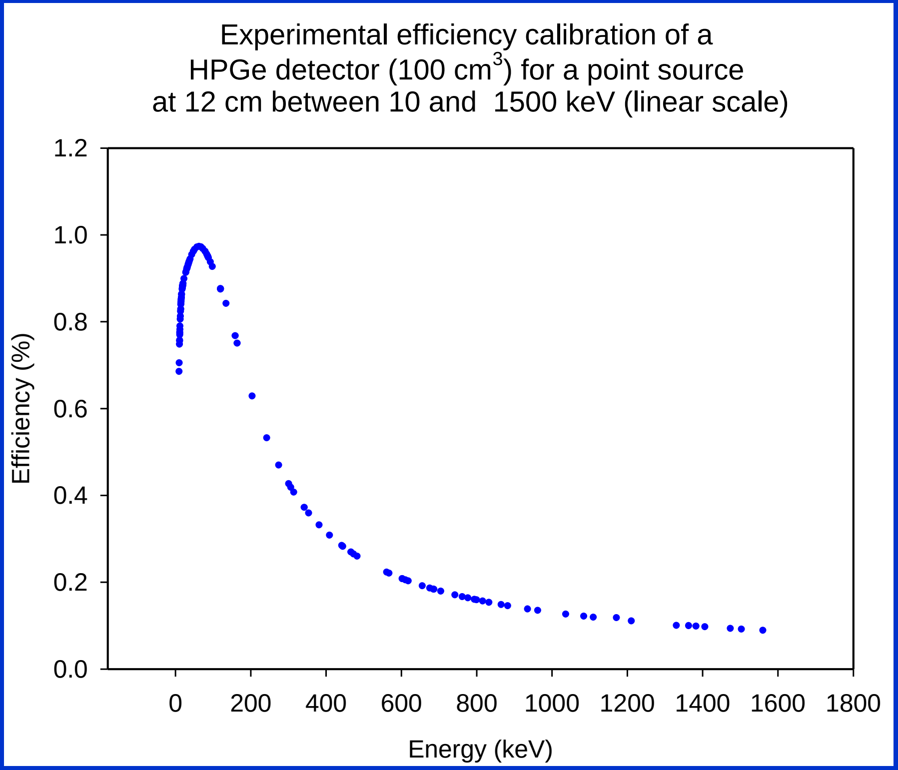
Other possible corrections

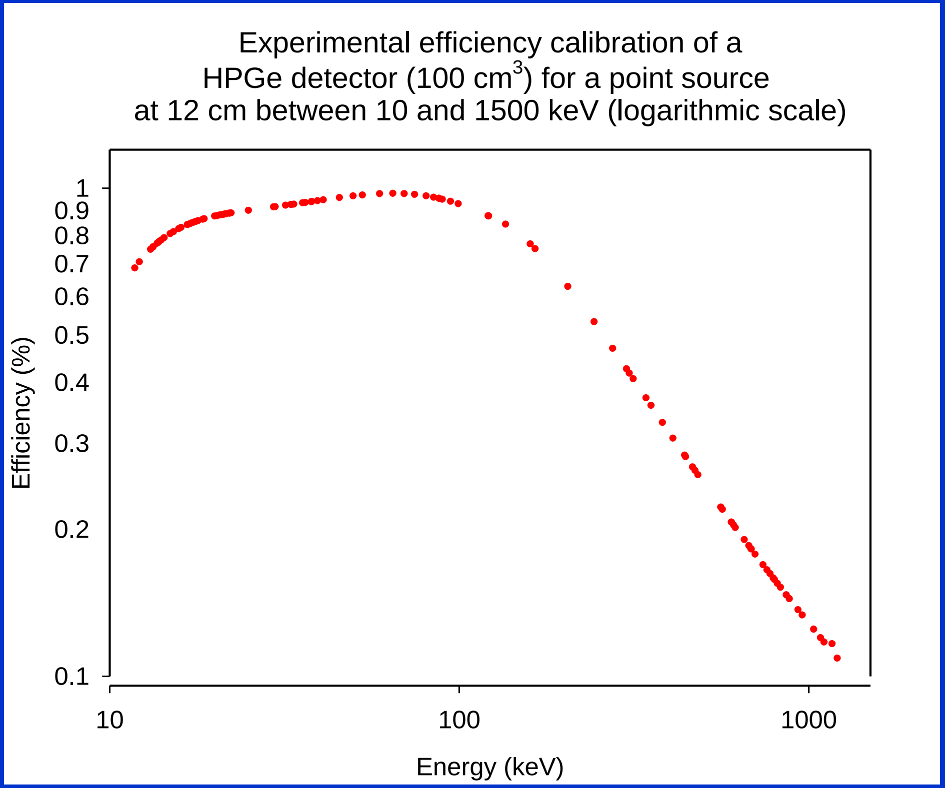
- Escape peaks
  - Annihilation (-511 and – 1022 keV)
  - Ge K X-rays (-11keV)
- Background
  - Natural radionuclides
  - Variation of Rn content versus time
- Dead time ?(if the correction is not accurate depending on the counting rate)

EFFICIENCY CURVE

Efficiency calibration : mathematical fitting (1)

Determination of the best fitted function to a given set of experimental data (energy, efficiency)





In the logarithmic scale , the shape is smoother than in the linear scale.

Efficiency calibration : mathematical fitting

Functions frequently used:

Polynomial fitting in the log-log scale:

$$\ln \varepsilon(E) = \sum_{i=0}^{n} a_i \cdot (\ln E)^i$$

$$\ln \varepsilon(E) = \sum_{i=0}^{n} a_i \cdot E^{-i}$$

Remarks :

-a_i coefficients are determined using a least-squares fitting method

- experimental data must be weighted

- the polynomial degree (n) must be adjusted depending on the number of experimental data (p) : n << p

- in some case two different functions can be used with a cross point

- check the resulting fitted curves !

Efficiency calibration : mathematical fitting

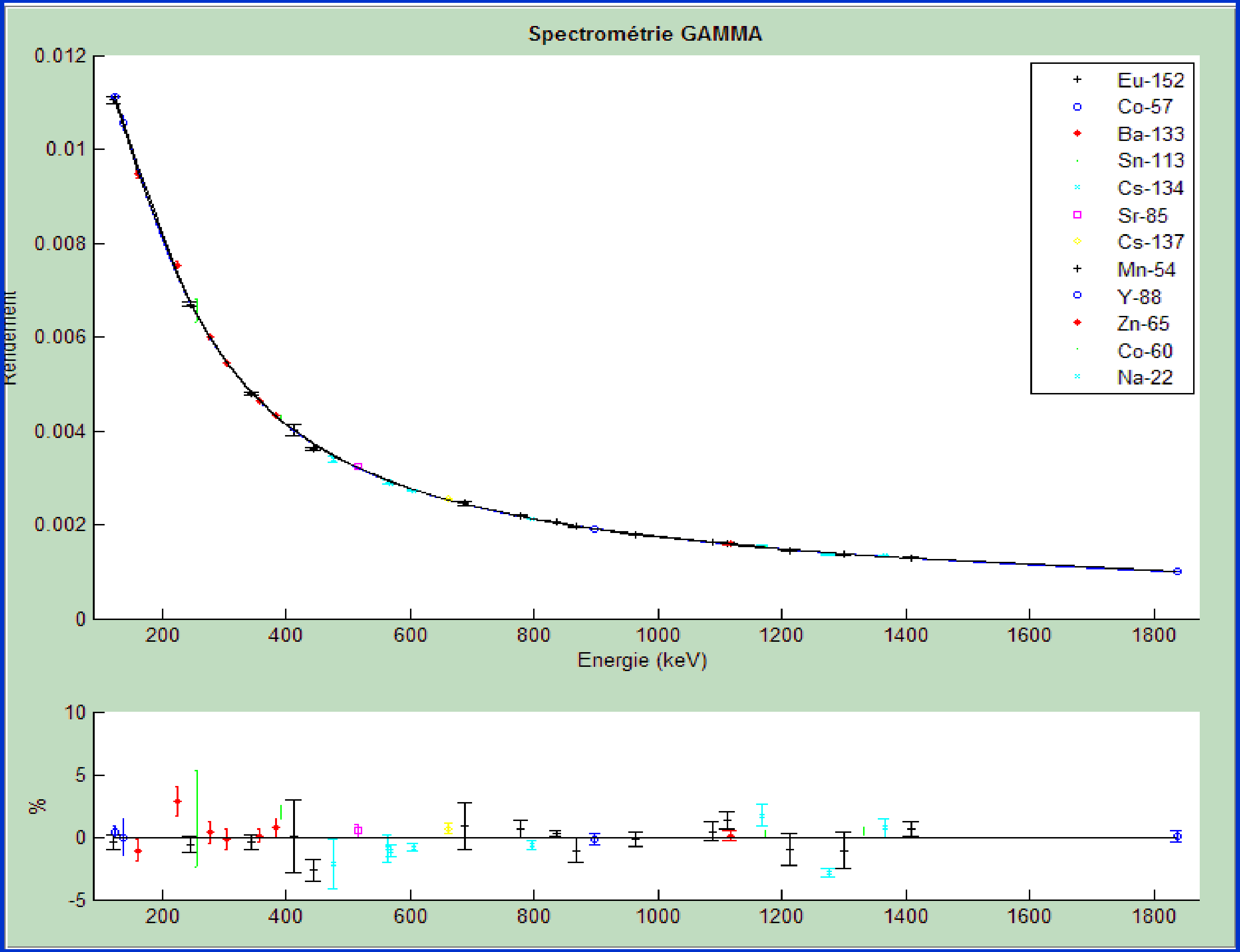
Example : 40 experimental values in the 122-to-1836 keV range

Fitting function :

$$\ln \varepsilon(E) = \sum_{i=0}^{n} a_i \cdot (\ln E)^i$$

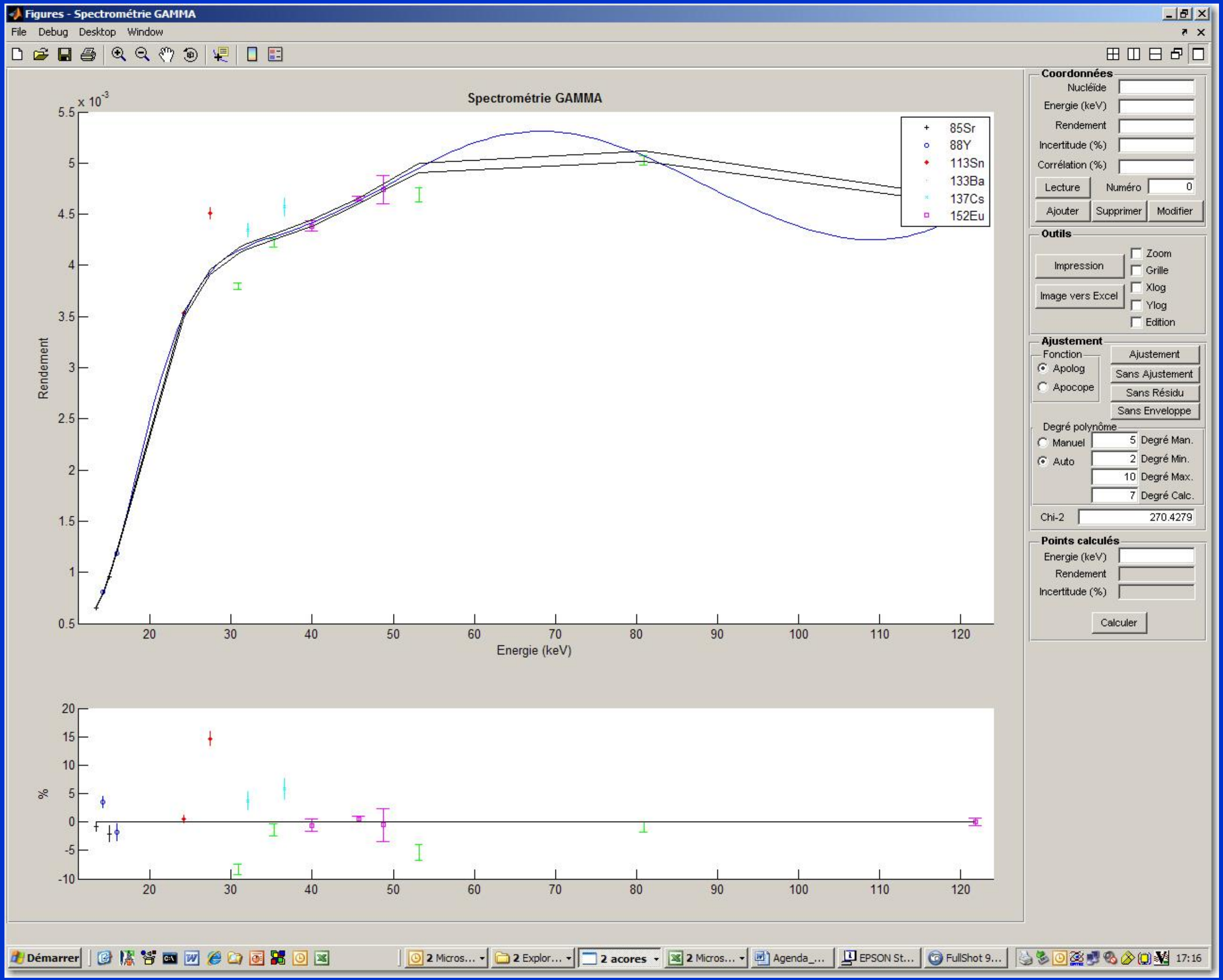
Adjusted coefficients :

fitting	122 to 1836 keV
deg0	-34,11961
deg1	48,16797
deg2	-25,89215
deg3	5,80219
deg4	-0,40503
deg5	-0,01632



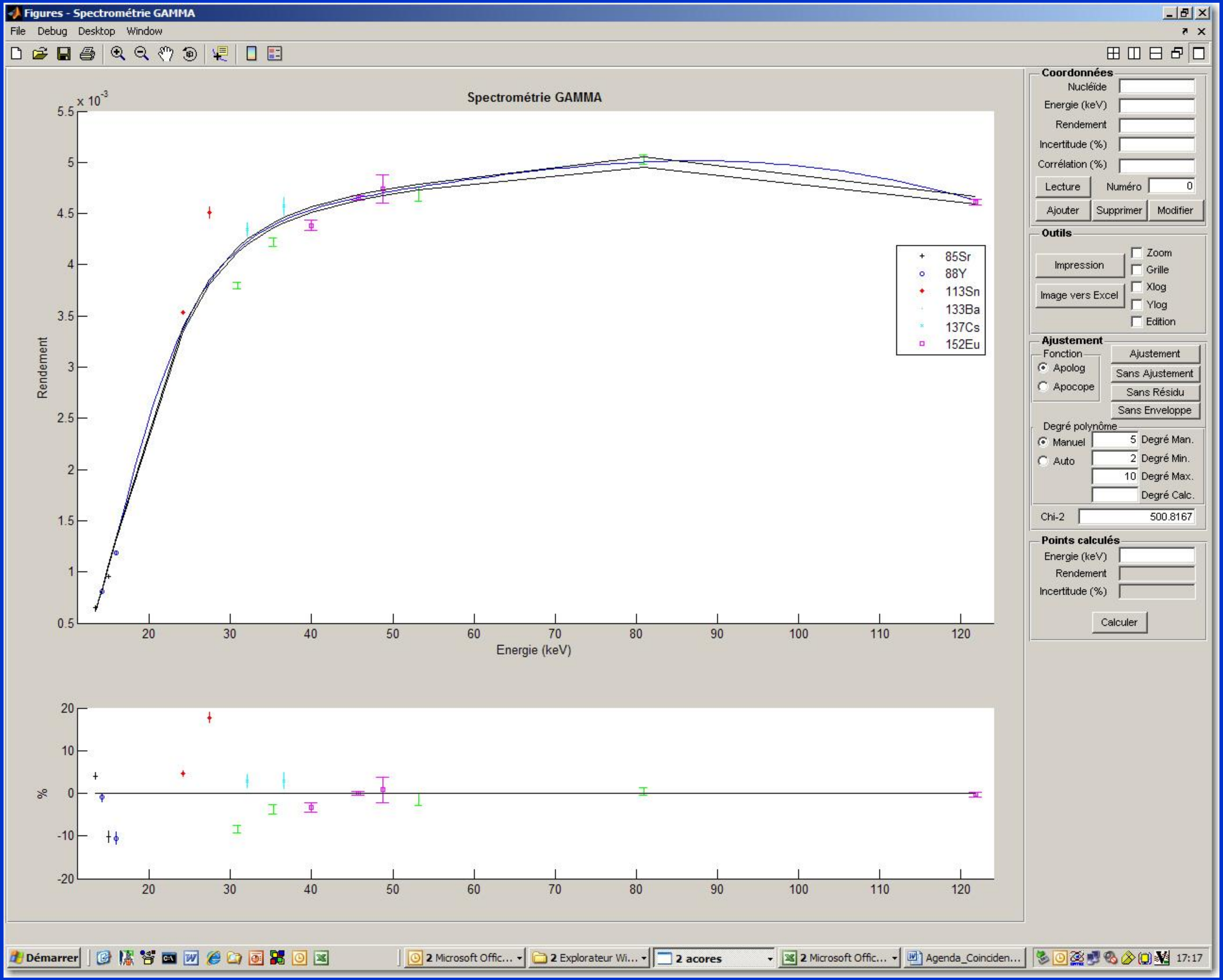
Efficiency calibration : mathematical fitting

Efficiency fitting must be visually checked



In the case of cross points be carefull :

- Avoid zones with important inflexion
- Avoid high degree polynomials



Uncertainty on the fitted efficiency

- The individual points have uncertainties
- The mathematical fitting can result in lower uncertainties
- Some correlations exits
  - Input data : one radionuclide- several energies
  - Calibration procedures,
  - etc.
- Carful examination is necessary

Conclusions

- Uncertainties are generally underestimated.
- Important to take each component into consideration.
- Corrective factors should be as close to 1 as possible (experimental conditions).