

*International Atomic Energy Agency Technical Visit on
Coincidence summing and geometry correction in gamma spectrometry*

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**Session 4 – Geometry corrections:
Applications**

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Geometry corrections: applications

1. Definition of full energy peak efficiency (FEPE)
 - Measurement of FEPE
 - Computation of FEPE
 - Geometry effects
2. Measurement of the efficiency for point sources
3. Measurement of the efficiency for small volume sources
 - Closed end coaxial detectors
 - Well-type detectors
4. Measurement of the efficiency for 100 – 3000 cm³ volume sources
5. Measurement of the efficiency for big volume sources
6. Summary

1. Definition of full energy peak efficiency (FEPE)

Ideally the full energy peak of energy E is defined as the part of the spectrum that corresponds to the complete absorption of the photons of energy E in the detector.

$$ED = E$$

-The full energy peak efficiency then should be the ratio of the number of events when the complete energy E was deposited in the detector to the number of photons of energy E emitted from the source

-Problem: the energy deposited can not be directly measured!

-The signals in the spectrum corresponding to a given energy deposited ED are distributed in a finite width of a peak with a given shape (fluctuations in the number of charge carriers, charge collection, electronic noise)

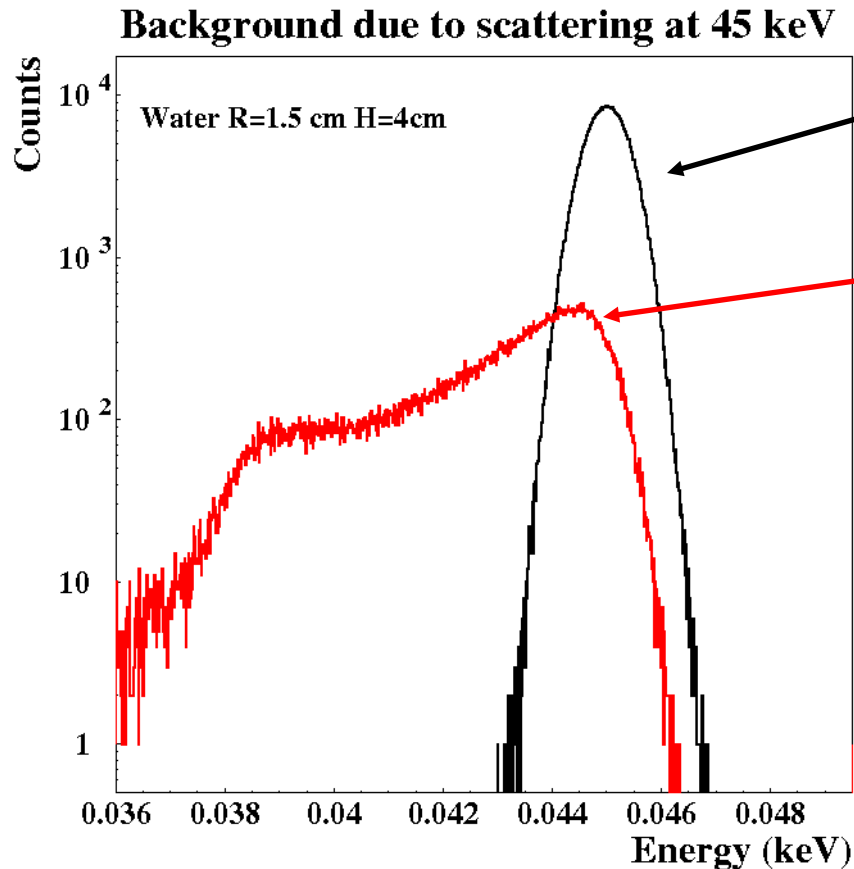
-The peak is superimposed over a background

-Part of the background can be due to the photons that deposit an energy smaller than E by a very small amount.

Monte Carlo simulation

Photons that deposited the complete energy E in the detector

Photons that were scattered in the source by low angle Compton effect. The energy deposit $ED < E$ but the signal is included in the peak



How is the FEPE measured?

Debertin and Helmer (Gamma- and X-ray spectrometry with semiconductor detectors, 1988, pag. 206):

$$\varepsilon(E) = \frac{n(E)}{R(E)}$$

$\varepsilon(E)$ the full energy peak efficiency

$n(E)$ the count rate in the peak of energy E

$R(E)$ the rate at which the photons of energy E are emitted from the source

The efficiency “**is related to a specific source-detector geometry and a particular peak analysis procedure; it is not a property of the detector**”

-many different peak analysis procedures exist

-Simple summation of the counts in a given limit

-Simple fit of a single peak

-Decomposition of multiplets

-**Important** when sample is analyzed on the basis of a calibration standard:

⇒**how reproducible is the procedure** (assuming the calibration standard and the sample are analyzed with the same peak analysis procedure)

- frequently the peaks in the calibration spectrum and the peaks in the spectrum of the sample have different statistics;
 - Is the procedure sensitive to the shape of the peak?
 - How sensitive is to the background estimation? (background contribution may be different)

How is the FEPE computed?

-In Monte Carlo codes (Vidmar et al., ARI 66 (2008) 764):

1. By counting the events when the photons did not interact in other media except the active volume of the detector and were completely absorbed in the active volume of the detector
2. By simulating the spectrum of energy deposition in the detector and counting the events from the channel of the spectrum corresponding to the energy E
 - dependence on the width of the channel – no possibility to discriminate in the channel the events in which a small energy was lost due to interactions outside of the active volume of the detector from the events when the complete energy of the photon was detected;
 - Very small channel width not appropriate due to rounding errors

3. Simulation of the energy deposition ED and distortion of the signal from the spectrum according to a Gaussian distribution (Decombaz et al., NIM 312 (1992) 152):

- without detector resolution ED \rightarrow Ch₀ (channel of peak centroid)
- with finite detector resolution σ (ED): sample randomly the channel (Ch)

from the distribution:

$$\text{ED} \rightarrow \text{Ch, random value} \quad p(\text{Ch}) = \frac{1}{\sigma \cdot \sqrt{2\pi}} \cdot \exp\left[-\frac{(\text{Ch} - \text{Ch}_0)^2}{2 \cdot \sigma^2}\right]$$

\Rightarrow analyze the simulated spectrum and the measured spectrum with the same peak analysis procedure

Are the computations by methods 1 and 3 equivalent?

- Problems:

-Contribution of the coherent scattering - negligible

-Contribution of low angle Compton scattering outside of the detector at low energies (at low energy, weak dependence of scattered photon energy on angle e.g. 45 keV: all cases with scattering angle $<30^\circ$ have $E > 44.5$ keV)

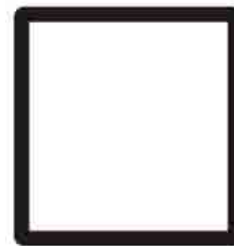
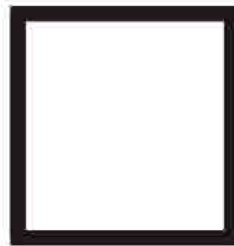
-scattering in the environment, in the sample (depends on the matrix of the sample), in the source support (point sources)

Source

Source



Scattering
medium

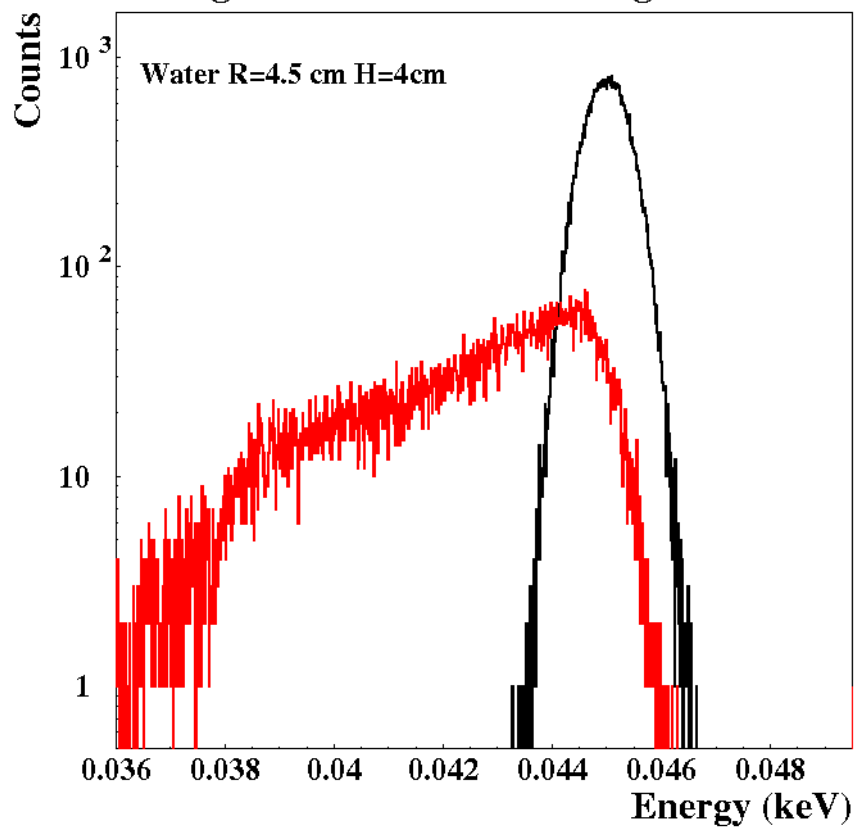


Detector

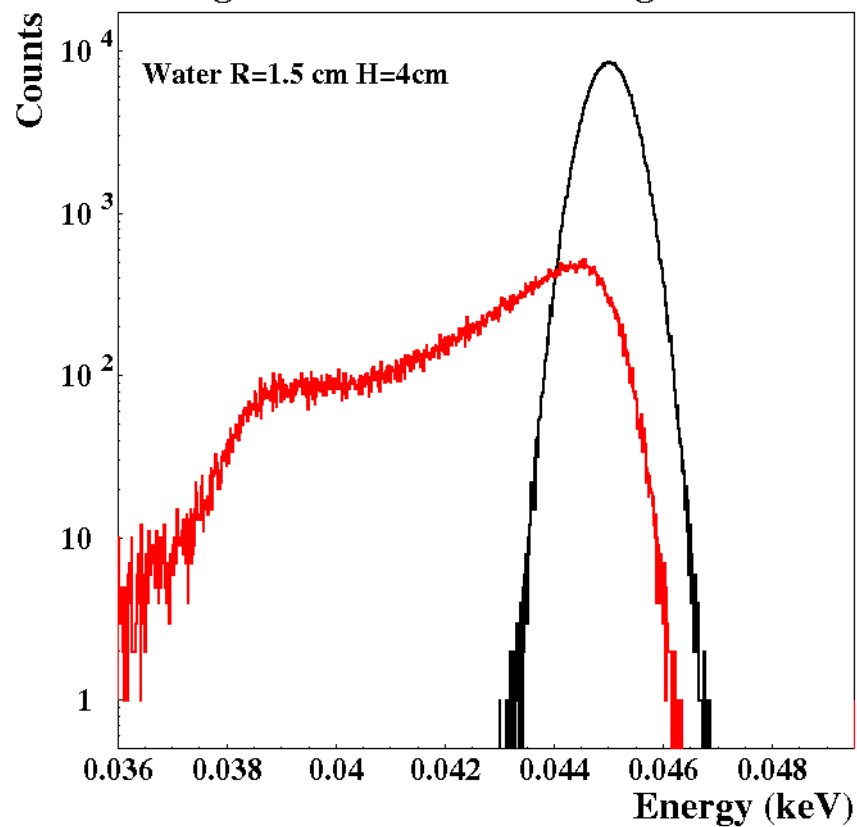
Detector

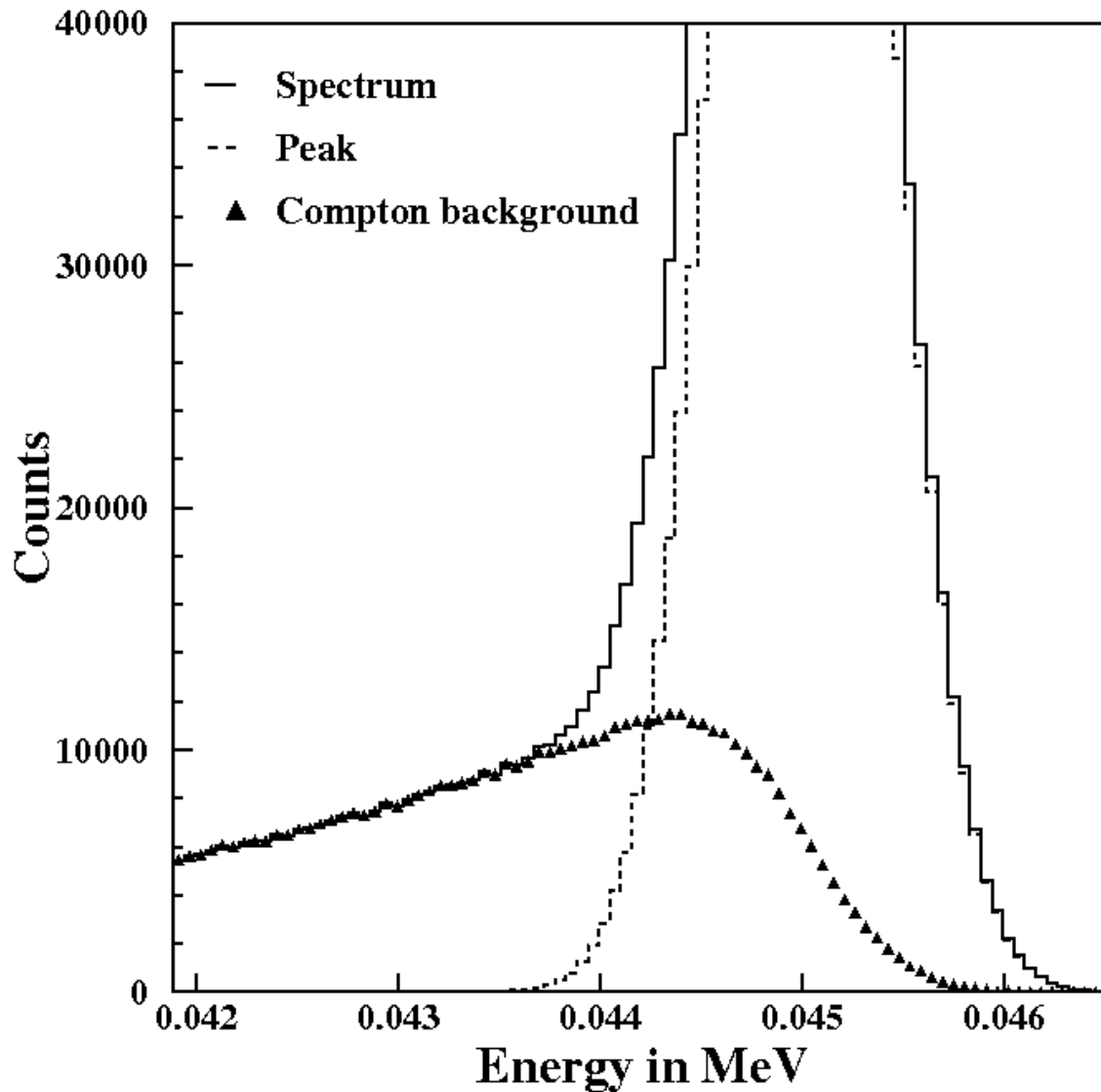
Due to small angle scattering in the environment, the peak count rate is increased. Should this increase be included in the peak efficiency?

Background due to scattering at 45 keV



Background due to scattering at 45 keV





Water sample
R=4.5 cm, H=4 cm

Compton contribution
under the peak of 45
keV: 13%

The linear
approximation or the
step approximation for
the background do not
remove completely this
contribution

Sima and Arnold, ARI
67 (2009) 701

-Differences in the definition of the measured FEPE (also between various peak analysis procedures) and in the definitions of the computed values of the FEPE
 \Rightarrow The effect of the differences almost cancels out if ratios of two efficiencies evaluated similarly are computed
 \Rightarrow Transfer method more robust and more reliable (Lepy et al., ARI 55 (2001) 493; Vidmar et al., ARI 66 (2008) 764).

Geometry effects

- Probability of a photon to be completely absorbed in the detector depends on the emission points

$$R(E) = \int_V dV \int_{\Omega} \exp[-\mu_m(E) \cdot l_m(\vec{r}, \vec{n})] \cdot T(E; \vec{r}, \vec{n}) \cdot p_i(E; \vec{r}, \vec{n}) \cdot \frac{A}{4\pi V} \cdot p(E) \cdot d\Omega$$

$R(E)$ – the count rate in the peak of energy E

A – the activity of the source, considered uniformly distributed within V

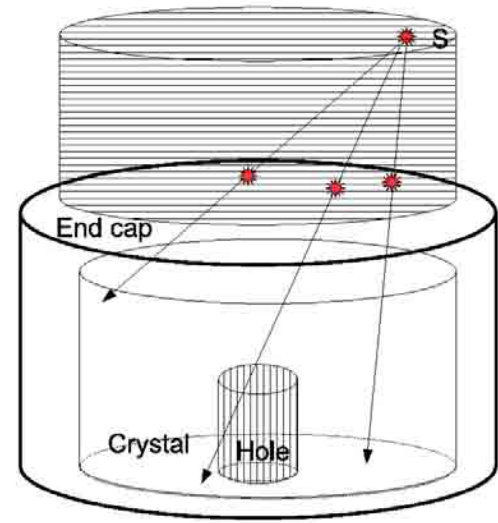
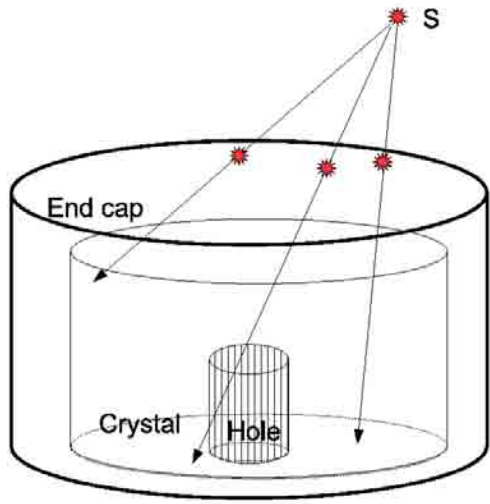
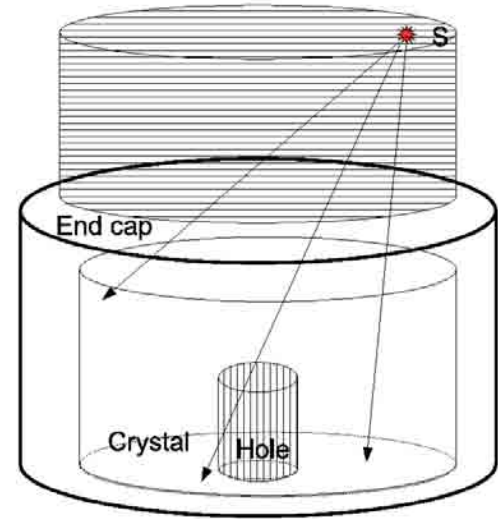
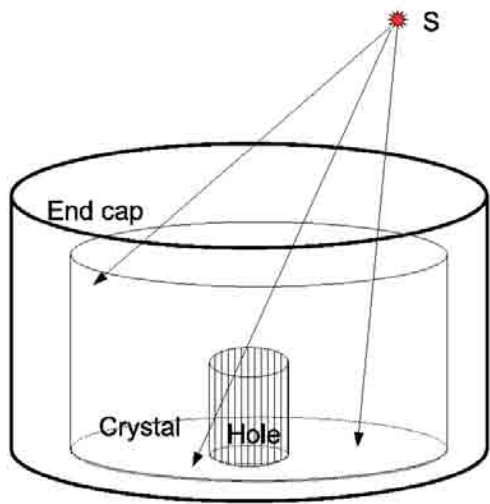
V – the volume of the source

$P(E)$ – the emission probability of a photon per decay

$\exp[-\mu_m(E) \cdot l_m(\vec{r}, \vec{n})]$ - probability of the transmission through the sample

$T(E; \vec{r}, \vec{n})$ - probability of transmission through the walls of the sample, absorbers

$p_i(E; \vec{r}, \vec{n})$ - probability of complete absorption if entered into the detector



$$\varepsilon(E) = \frac{1}{4\pi V} \int_V dV \int_{\Omega} \exp[-\mu_m(E) \cdot l_m(\vec{r}, \vec{n})] \cdot T(E; \vec{r}, \vec{n}) \cdot p_i(E; \vec{r}, \vec{n}) \cdot d\Omega = \frac{1}{V} \int_V \varepsilon^P(E; \vec{r}) \cdot dV$$

$\varepsilon^P(E; \vec{r})$ - The efficiency for an elementary point source located in the sample

Quasi-point source: the dependence of $\varepsilon^P(E; \vec{r})$ on the location of the emission point within the source volume negligible

$\Rightarrow \varepsilon^P(E; \vec{r}) \approx \varepsilon(E)$ for any r within the volume of the sample

Geometry effects result from the dependence of the integral over the volume of the source on the geometry

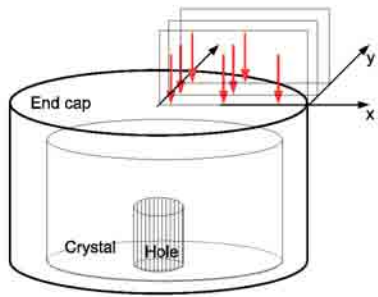
The difficult part of the computation – the p_i term

$$p_i(E; \vec{r}, \vec{n}) = 1 - \exp[-\mu_d(E) \cdot l_d(\vec{r}, \vec{n})] \quad ???$$

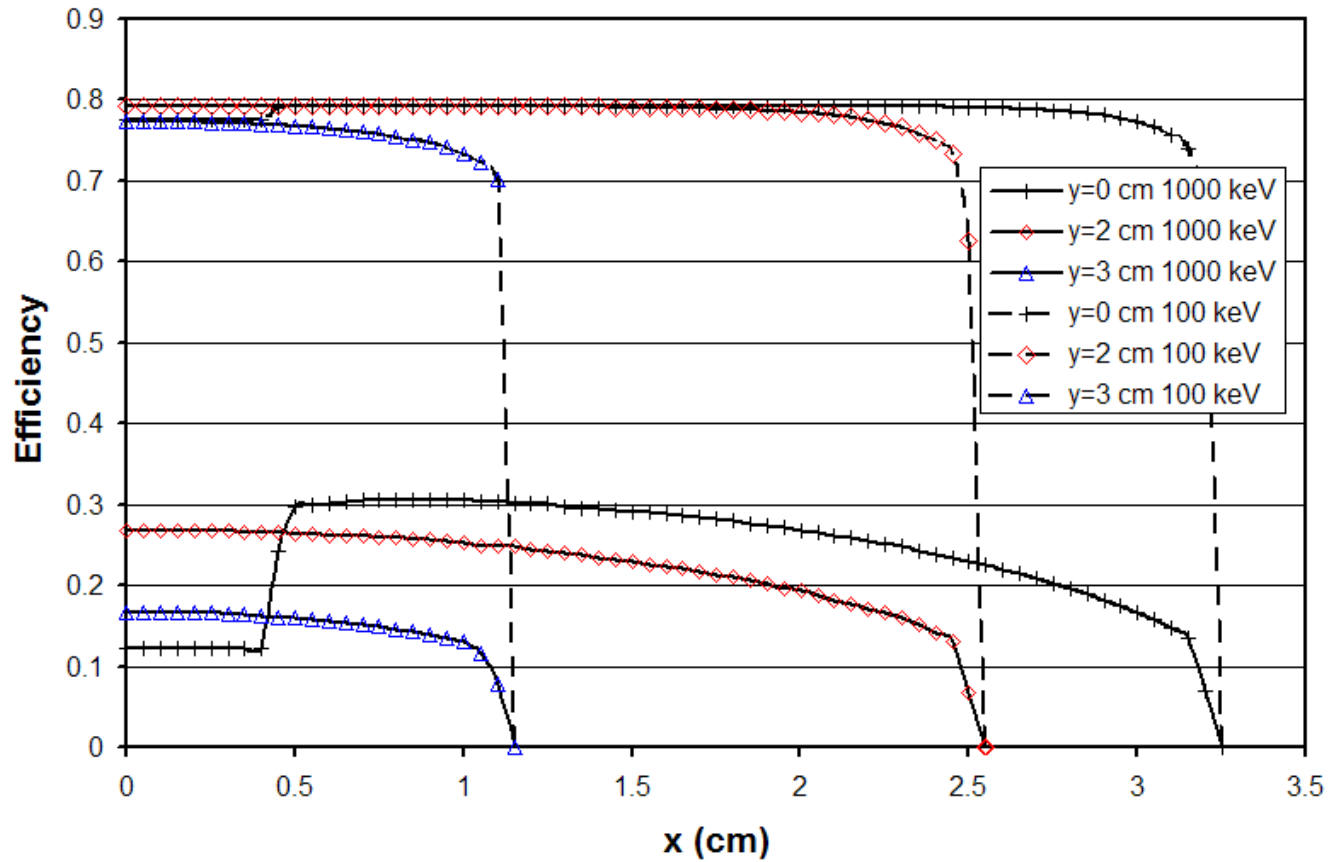
Low energies - OK

Energies higher than 50 keV – not OK, contribution of multiple scattering

Selim et al., Rad. Phys. Chem 48 (1996) 23; Abbas, ARI 54 (2001) 761 – the “peak attenuation coefficient” – insufficiently founded



Photons perpendicular on the entrance window



2. Measurement of the efficiency for point sources

Use of standard sources traceable to NMI

Single gamma nuclides + Co-60 + Y-88

-Correct for dead time

-Correct for decay time

Uncertainty – standard activity uncertainty, P_γ uncertainty



Uncertainty of the measured efficiency points – at best the source's uncertainty

Uncertainty of the values of the efficiency curve ?

-at energies above 200-300 keV – safe interpolation (but problems with Co-60, Y-88)

=> uncertainty probably 2 %

-around the maximum of the efficiency curve (80 – 180 keV)

=> higher efficiency, less measured efficiency points available

-correlations in the efficiency curve at different energies

=> complications in the evaluation of the uncertainties

Best accuracy

– 0.1 % for the ratio of the efficiency at E_1 to the efficiency at E_2 in the energy range 433 – 2754 keV (Luddington and Helmer, NIMA 446 (2000) 506)

- 0.2 % uncertainty of the absolute value of the efficiency from 50 to 1400 keV (Helmer et al., NIMA 511 (2003) 360)

-0.4 % uncertainty of the absolute value of the efficiency from 50 to 4800 keV (Helmer et al., ARI 60 (2004) 173).

Use of point source measurements for mapping the point source efficiency close to the detector – position of the source very important

3. Measurement of the efficiency for small volume sources

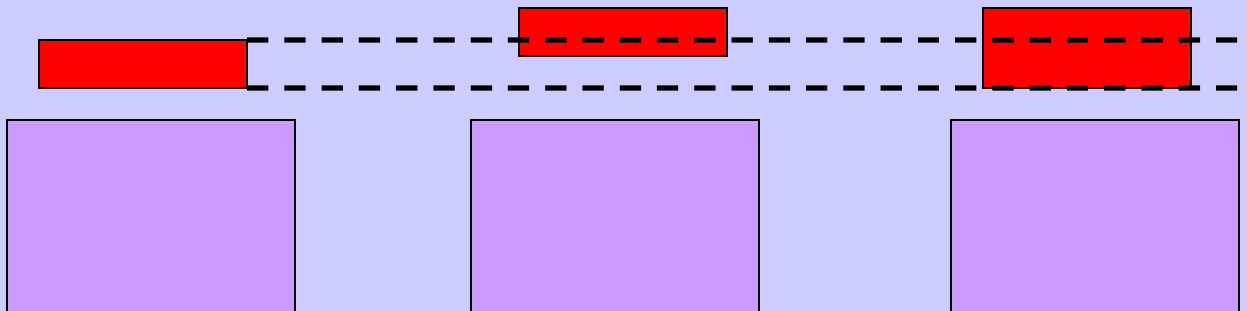
Small volume sources

– self-attenuation not important

- elementary point source efficiency dependence on the position of the emission point inside the volume of the source not negligible

$\varepsilon^P(E; \vec{r})$ not constant over the volume of the sample

Variation due especially to solid angle variation; source close to the detector



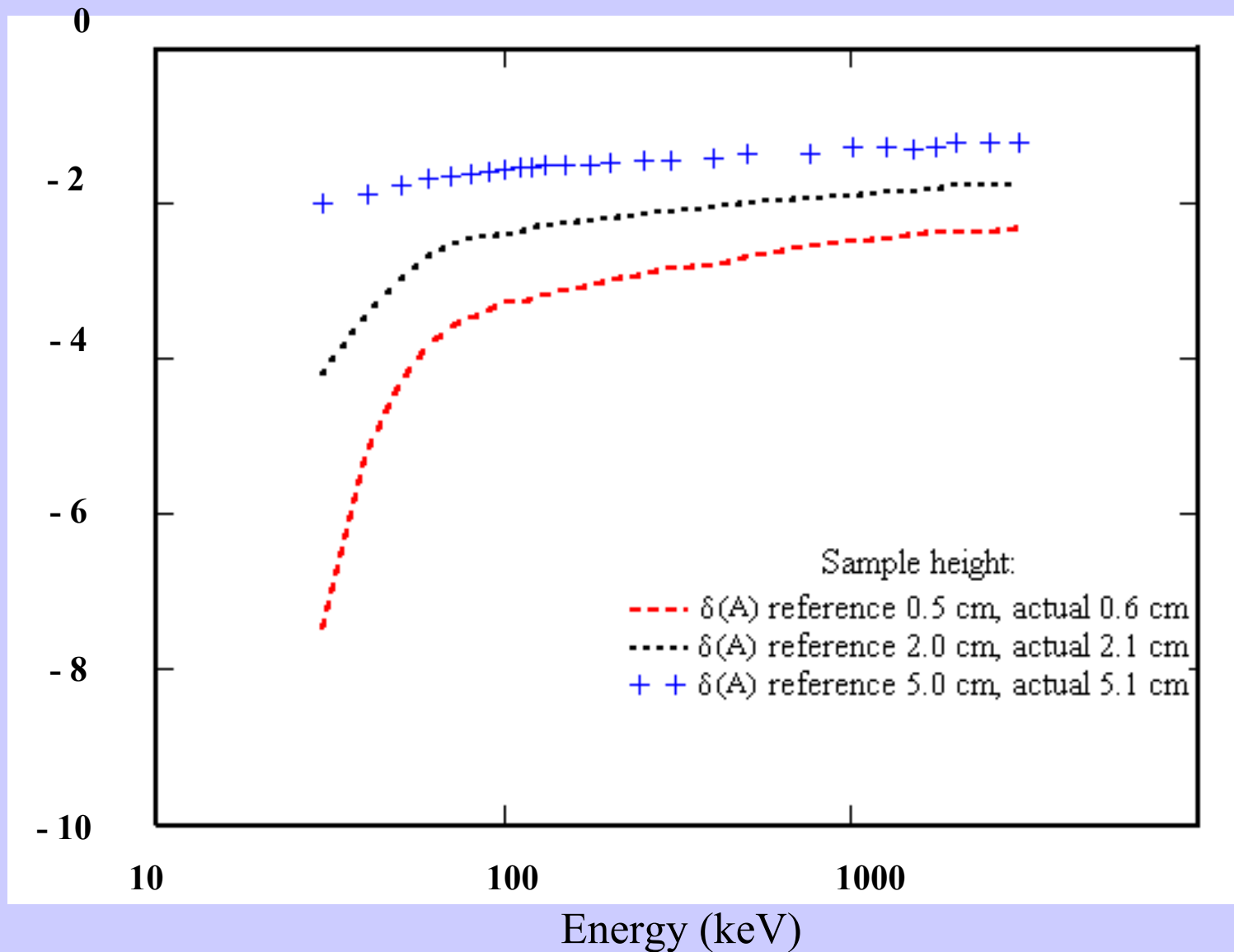
-geometry effects more important

-If the sample and the standard are not exactly the same

⇒Keep the mass the same?

⇒Keep the height the same? => much better

Relative error of activity $\delta(A)$ (%) when the reference has a height less by 1 mm than the sample. Soil sample, $R=3.5$ cm.



⇒ High uncertainties due to the geometry

⇒ The efficiency varies strongly with the position, coincidence summing - stronger

⇒ **Solution**

⇒ **Well-type detector**

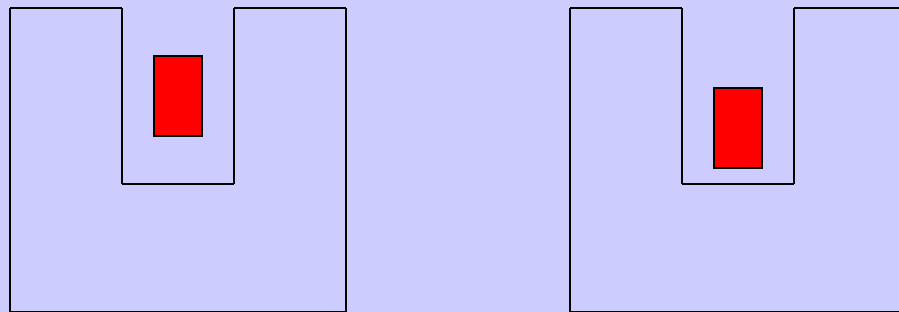
- the change in the solid angle very small

- insensitive to the position of the sample

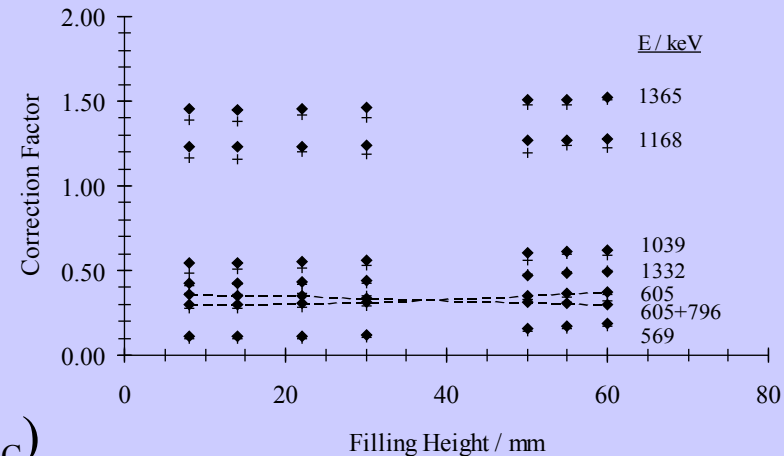
- the detector can be lined with absorbers to reduce coincidences with

X-rays

Gelsema and Blaauw, NIMA 368 (1996) 410



Small sensitivity to geometry (efficiency and F_C)



Sima and Arnold, ARI 47 (1996) 889

Both closed end coaxial detectors and well-type detectors:

=> Uncertainty of the values of the measured efficiency – similar consideration as for point sources (dependence on the energy range) (Assuming no geometry induced uncertainty)

4. Measurement of the efficiency for 100 – 3000 cm³ volume sources

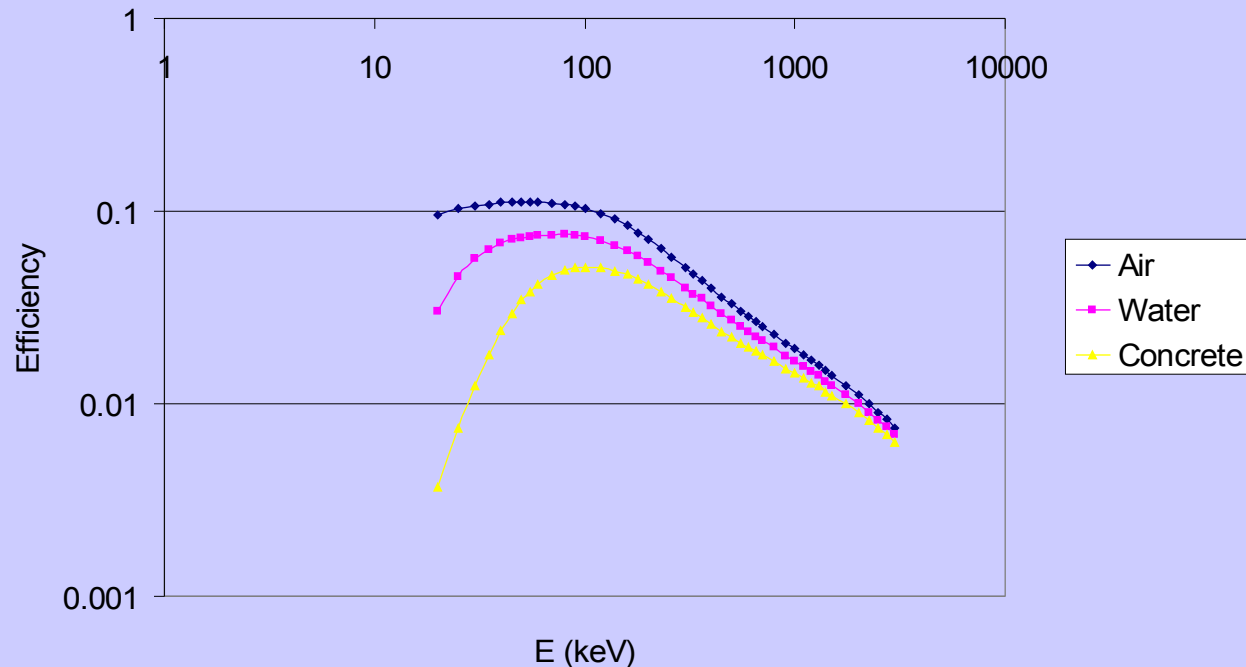
Both geometry and attenuation effects are important

Efficiency curve for the standard samples – interpolation for other energies

Cylindrical samples, H=5 cm

Efficiency around the maximum – depends on sample matrix

=> the limit between the low and high energy of the efficiency curves specific to the sample matrix



Spiked samples

Homogeneity – both for standard, for spiked sources and for measured samples

Test

-measurements using symmetry

-Use both gamma transition peaks and sum peaks when available

$$R_0(E) = \int_V S_0 \cdot \varepsilon^{(p)}(E; \vec{r}) \cdot I(E) \cdot dV \quad \text{Count rate for uniform source} \quad S(\vec{r}) = S_0 = \frac{A}{V}$$

$$R(E) - R_0(E) = \int_V [S(\vec{r}) - S_0] \cdot \varepsilon^{(p)}(E; \vec{r}) \cdot I(E) \cdot dV \quad \text{Normal peak, non-uniform}$$

$$R^{SumPeak}(E_1 + E_2) - R_0^{SumPeak}(E_1 + E_2) = \int_V [S(\vec{r}) - S_0] \cdot \varepsilon^{(p)}(E_1; \vec{r}) \cdot \varepsilon^{(p)}(E_2; \vec{r}) \cdot I(E_1, E_2) \cdot dV$$

If the efficiency is constant, no possibility to check homogeneity, S_0 and $S(r)$ will give the same count rate; normal peaks and sum peaks sample differently the non-homogeneity of the sample

=> Simply adding an uncertainty due to the non-homogeneity of a CRM not a solution; the effect of non-homogeneity is different for different peaks of the same nuclide. Correct evaluation of the sample only if the computation uses exactly the same peak as when the effect of non-homogeneity was evaluated

Efficiency transfer

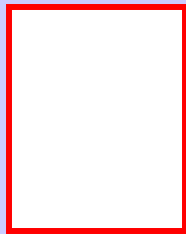
-Similar samples – GESPECOR

=> Correlated sampling – good uncertainty

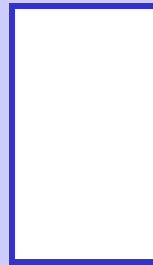
Sima and Arnold, ARI 56 (2002) 764

Efficiency transfer for volume sources with slightly different geometry

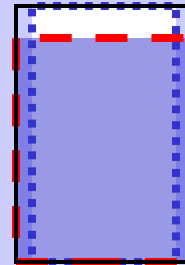
- actual geometry slightly different from the reference geometry
 - identical containers but different filling height
- Transfer factor:
 - Monte Carlo simulation based on a correlated sampling technique
 - simultaneous computation of each efficiency



Actual geometry



Reference

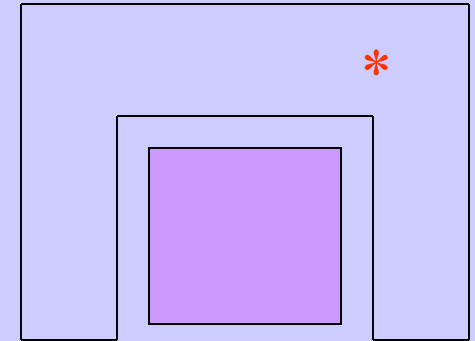


Simulation

Transfer factor as close to 1 as possible:

-Method of the representative point (Jun Saegusa, ARI 66 (2008) 774)

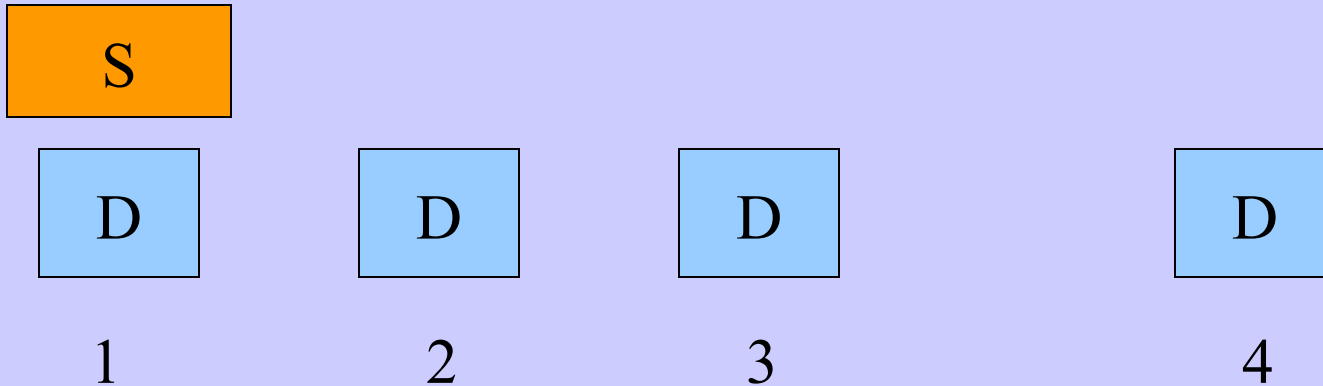
-Point source efficiency close to the efficiency for the volume source



Sources bigger than the CRM available

In the case of vacuum sources the count-rate (CR) for the big source satisfies:

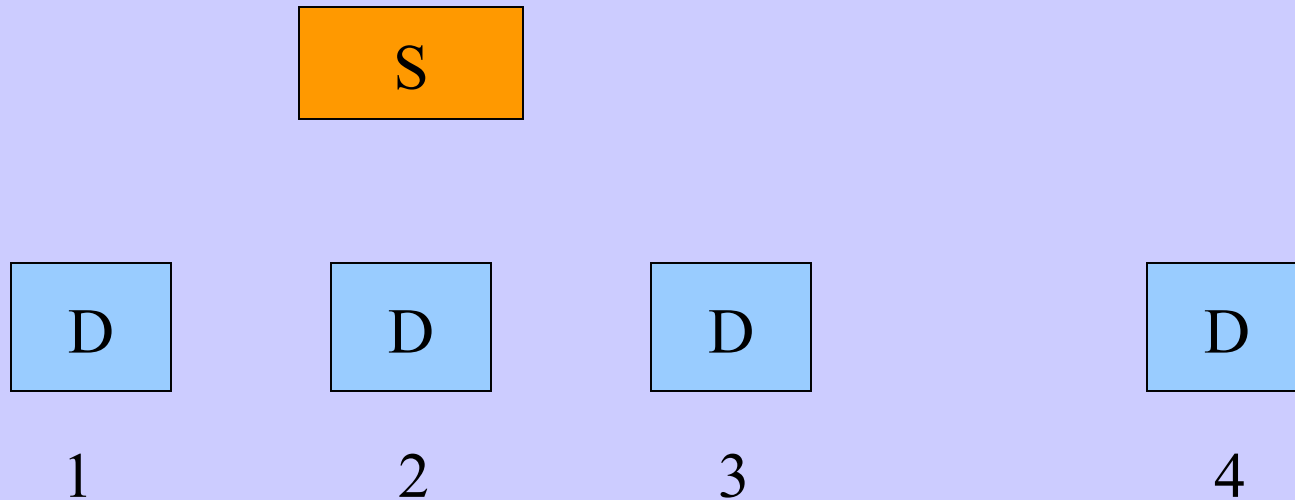
$$CR_4(3S) = CR_1(S) + CR_2(S) + CR_3(S)$$



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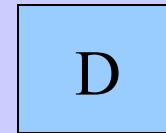
1



2



3

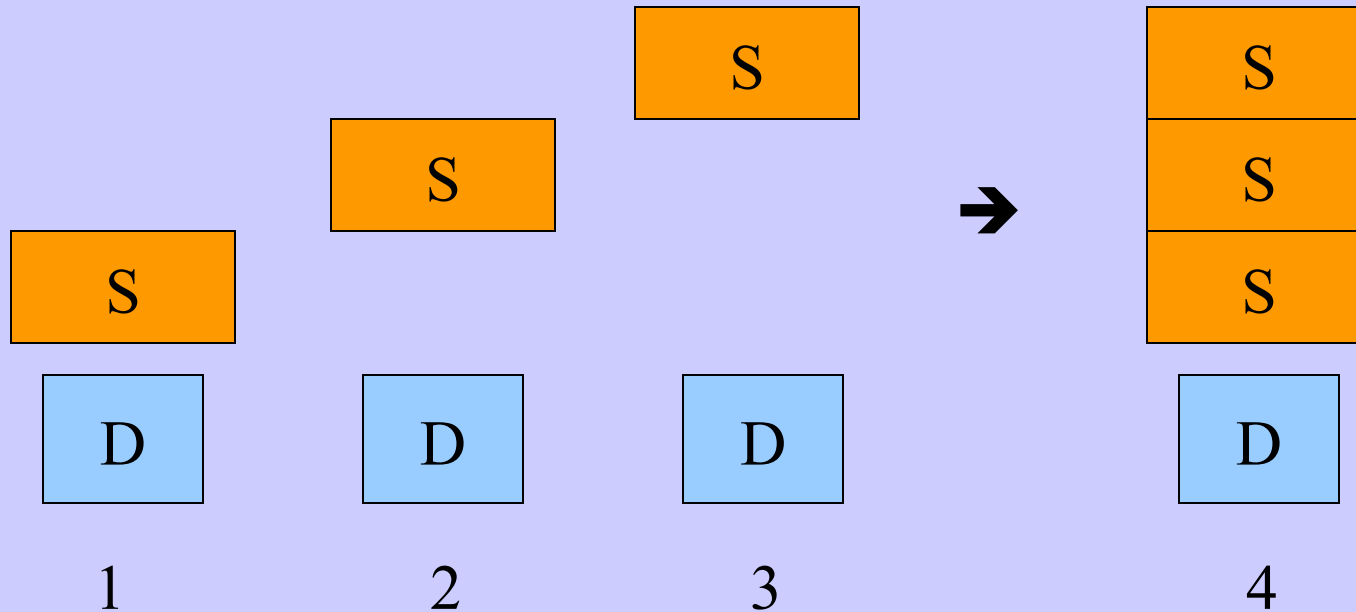


4

Sources bigger than the CRM available

In the case of vacuum sources the count-rate (CR) for the big source satisfies:

$$CR_4(3S) = CR_1(S) + CR_2(S) + CR_3(S)$$



⇒ Correct the effects of self-attenuation: $\varepsilon_i(E; 0) = F_{ai}(E; 0; m) \varepsilon_i(E; m)$

⇒ Linear relations between the values of efficiency in geometry 4 and the efficiencies in geometries 1, 2 and 3 with reliable coefficients

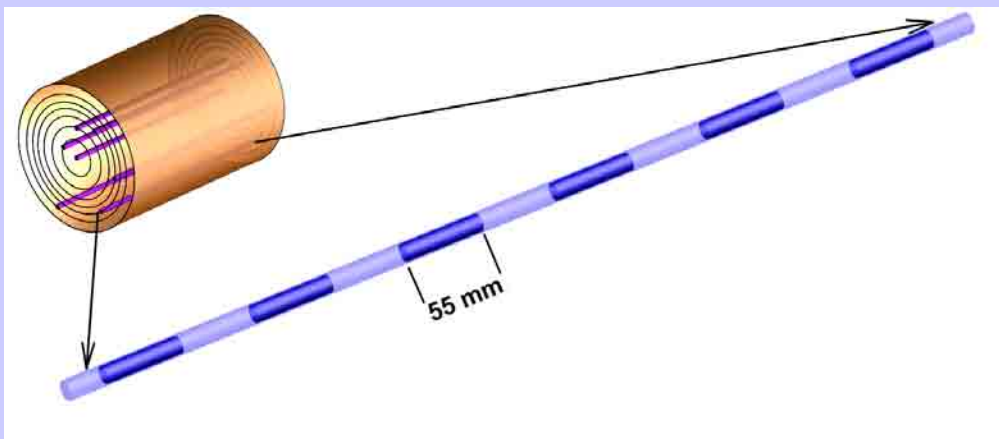
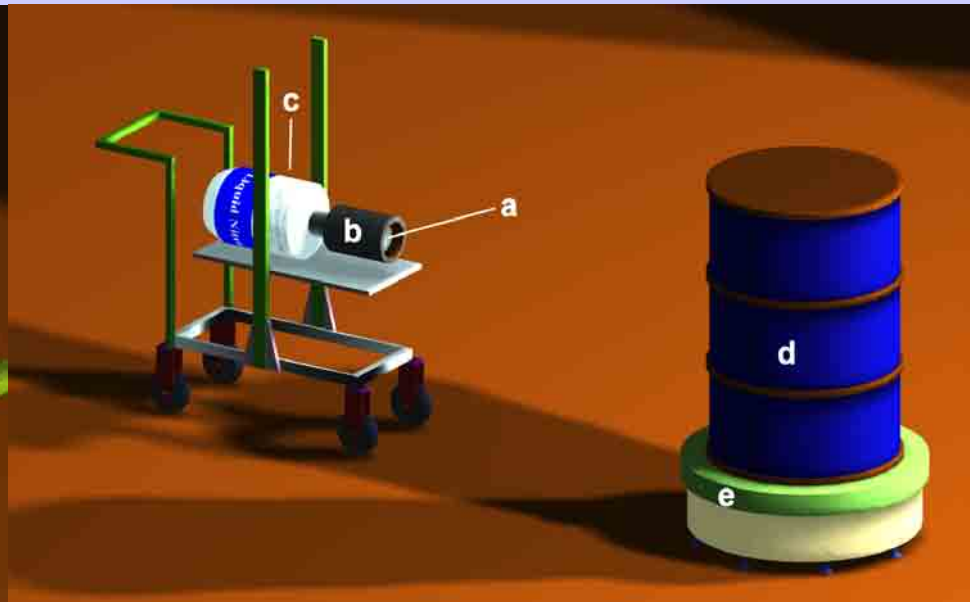
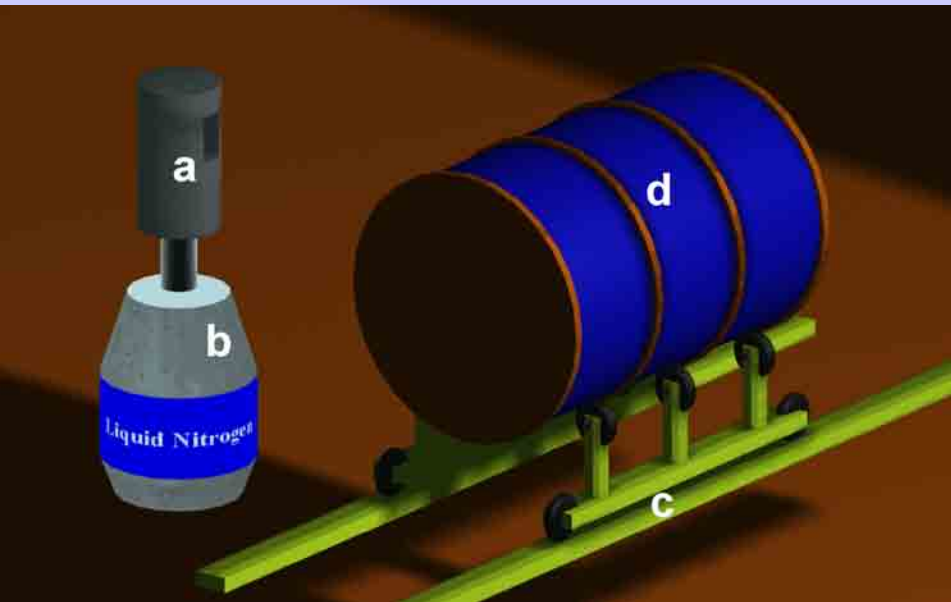
$$\varepsilon_v(E; 0) = [\varepsilon_1(E; 0) V_1 + \varepsilon_2(E; 0) V_2 + \varepsilon_3(E; 0) V_3] / V, \text{ with } V = V_1 + V_2 + V_3$$

$$\varepsilon_v(E; m) = F_{av}(E; m; 0) \varepsilon_v(E; 0)$$

5. Measurement of the efficiency for big volume sources

Waste drums of 200 l

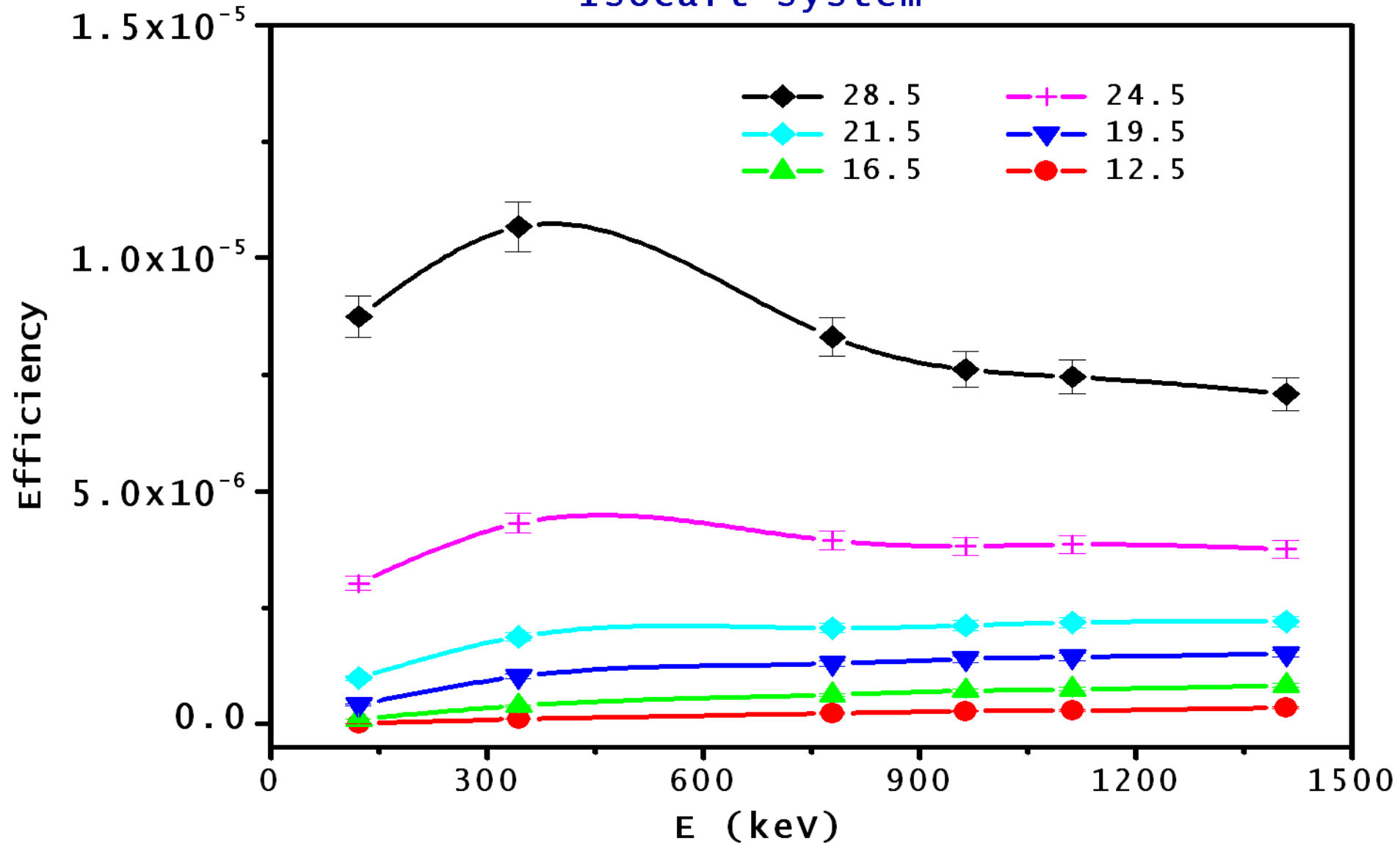
Experimental calibration: shell sources method



Liang et al., ARI 47 (1996) 669
Sima et al., ARI 61 (2004) 123
Toma et al., NIMA 580 (2007) 391

Stanga et al., ARI 68 (2010) 1418

Isocart system



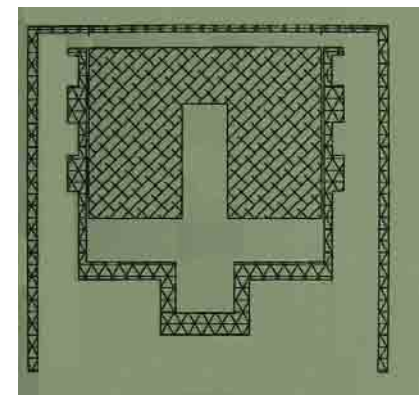
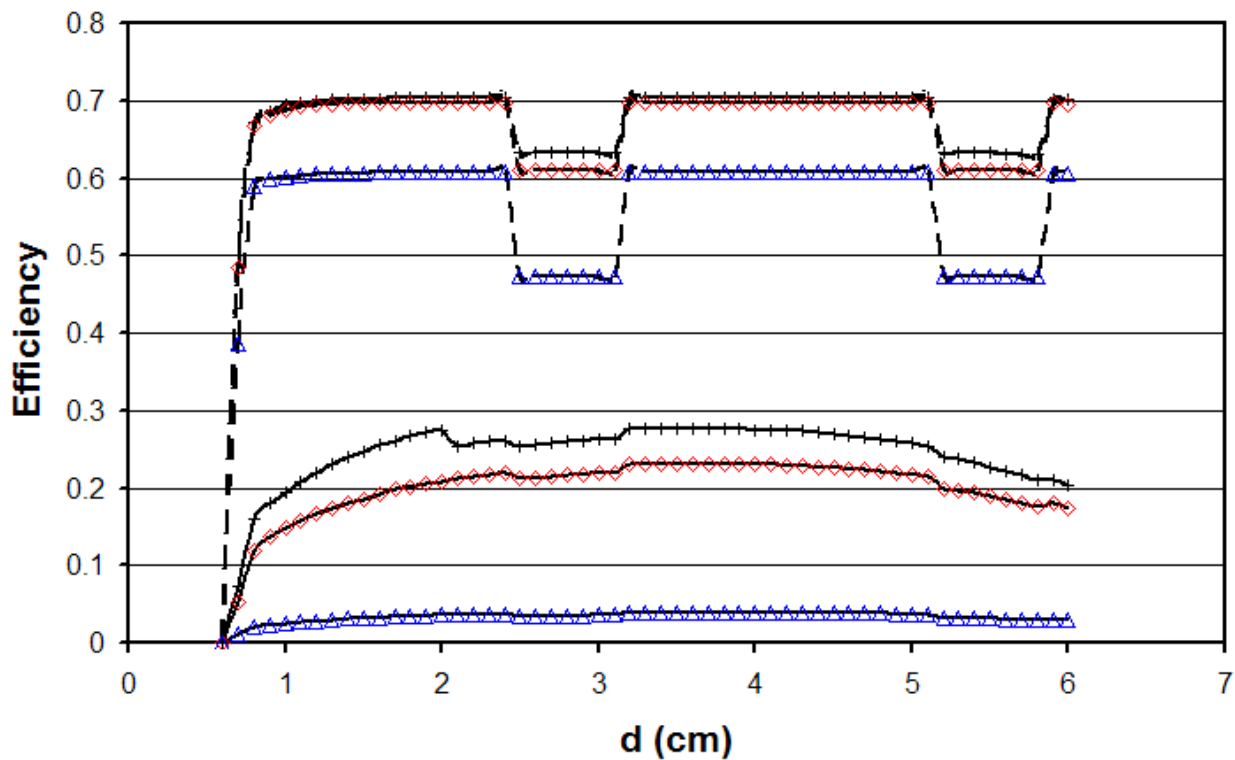
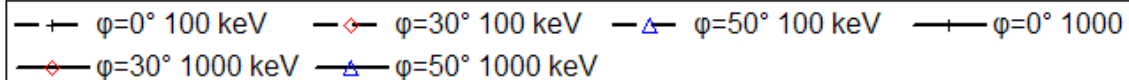
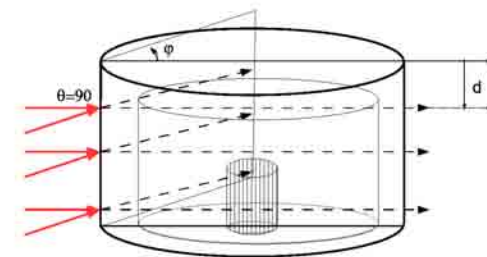
In all transfer methods it is better to have the geometry as close as possible, but also the detector response should be as much as possible the same (decrease also the statistical uncertainty)

⇒ Response function, applied for each computation

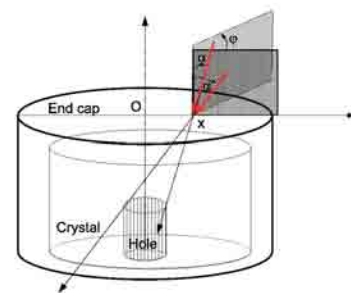
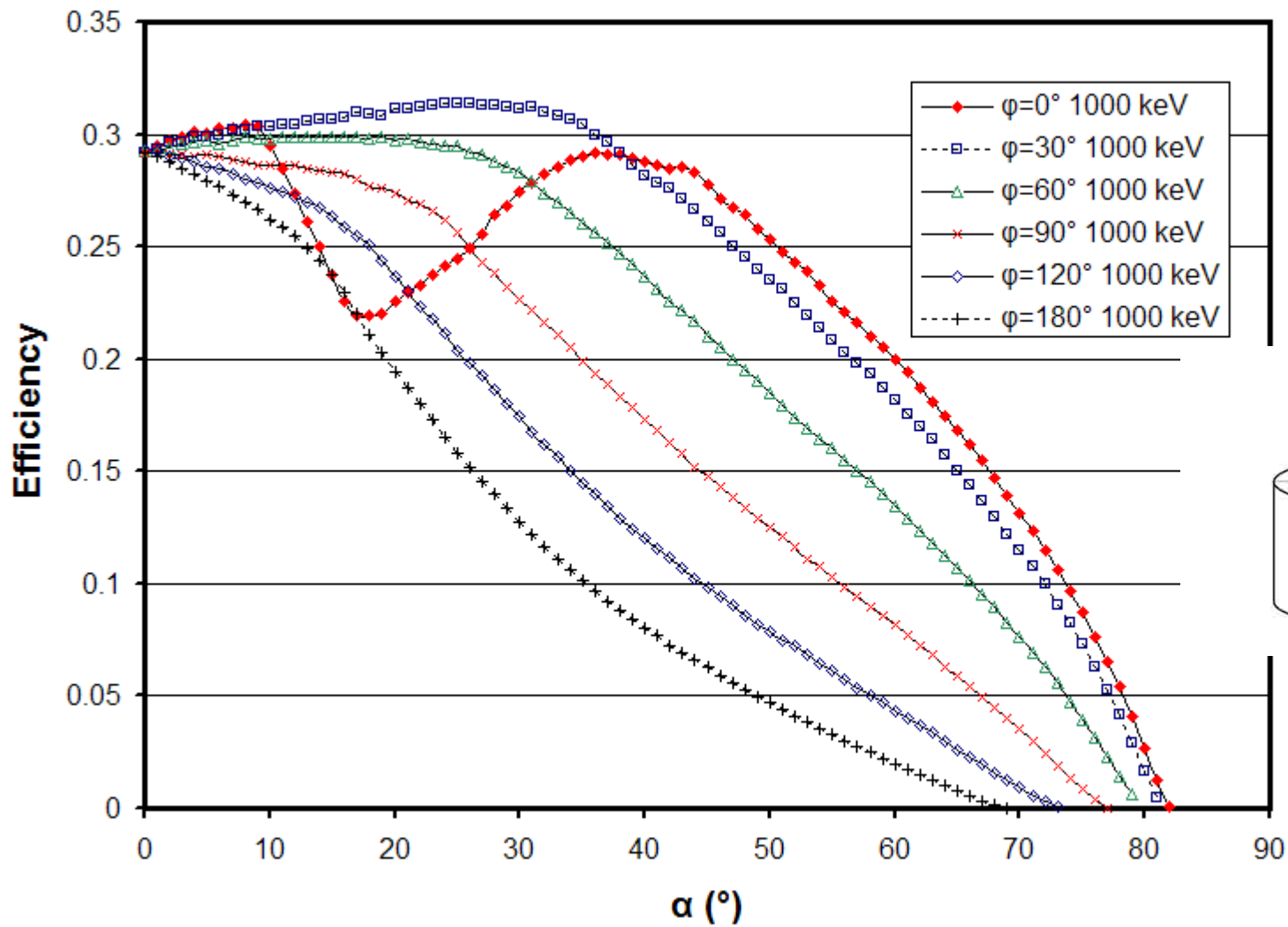
⇒ Advantage also from the point of view of time of Monte Carlo computation

Sima, ARI 68 (2010) 1403

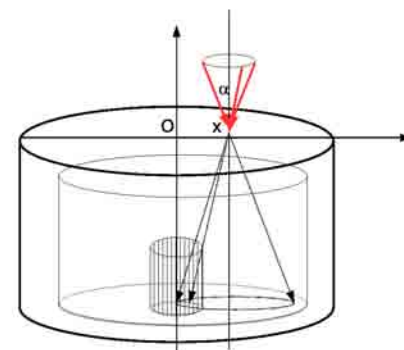
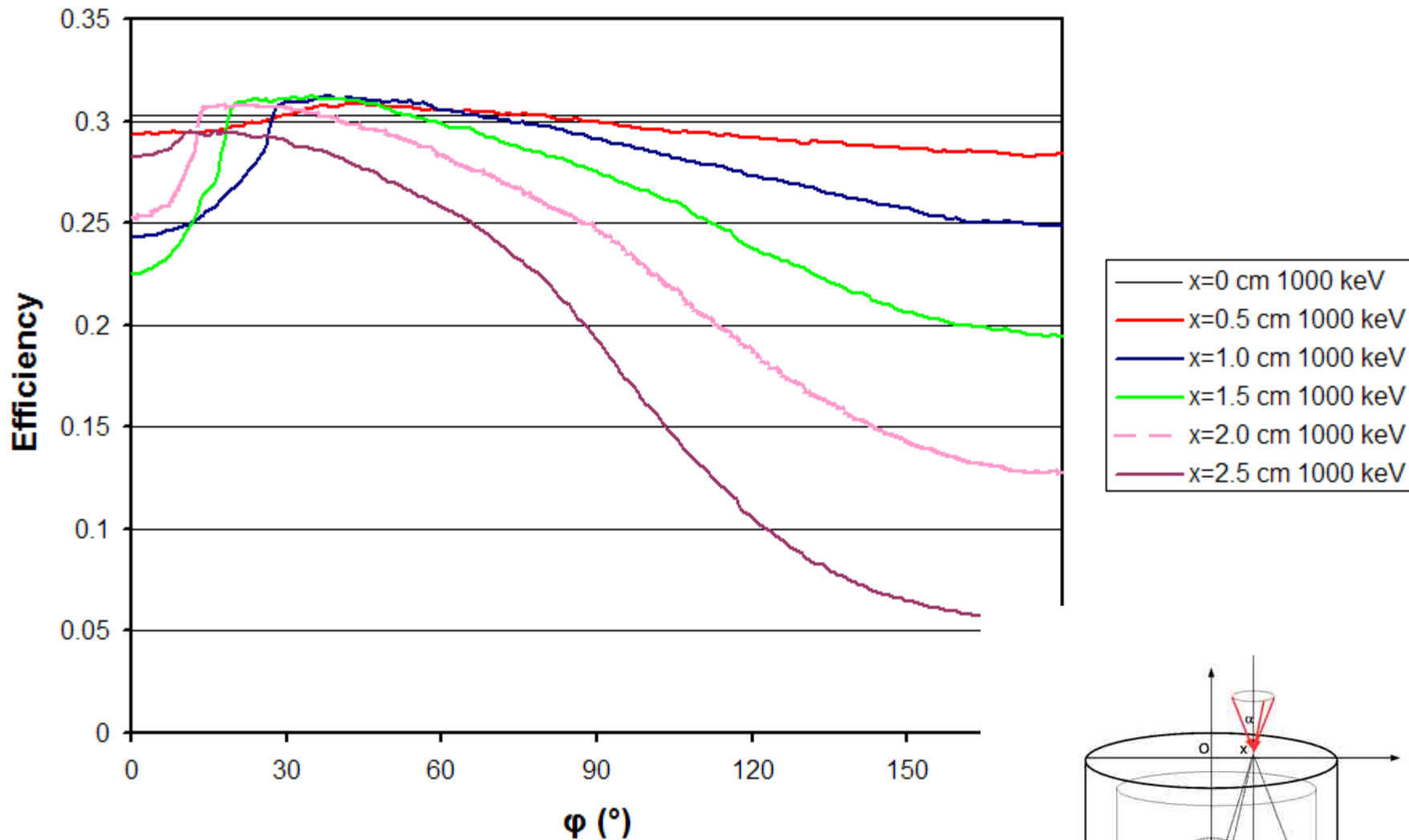
Photons perpendicular on the side of the end cap



X=1.5 cm 1000 keV



Photons with $E=1000$ keV incident on end cap at $\alpha=20^\circ$



6. Summary

- Experimental calibration precise, but possible only for specific nuclides and geometries
- Smaller uncertainty of the values of the efficiency directly measured
- Higher uncertainty of the calibration curve
- Efficiency for samples for which no standard exists – higher uncertainty
- Transfer method very useful
- Best results with transfer method when the two configurations are similar