Improving the accuracy of uncertainty calculations with Genie 4.0

Eric TISCHENBACH
Application support manager
Europe, Africa, Middle East & Asia

28 September 2023

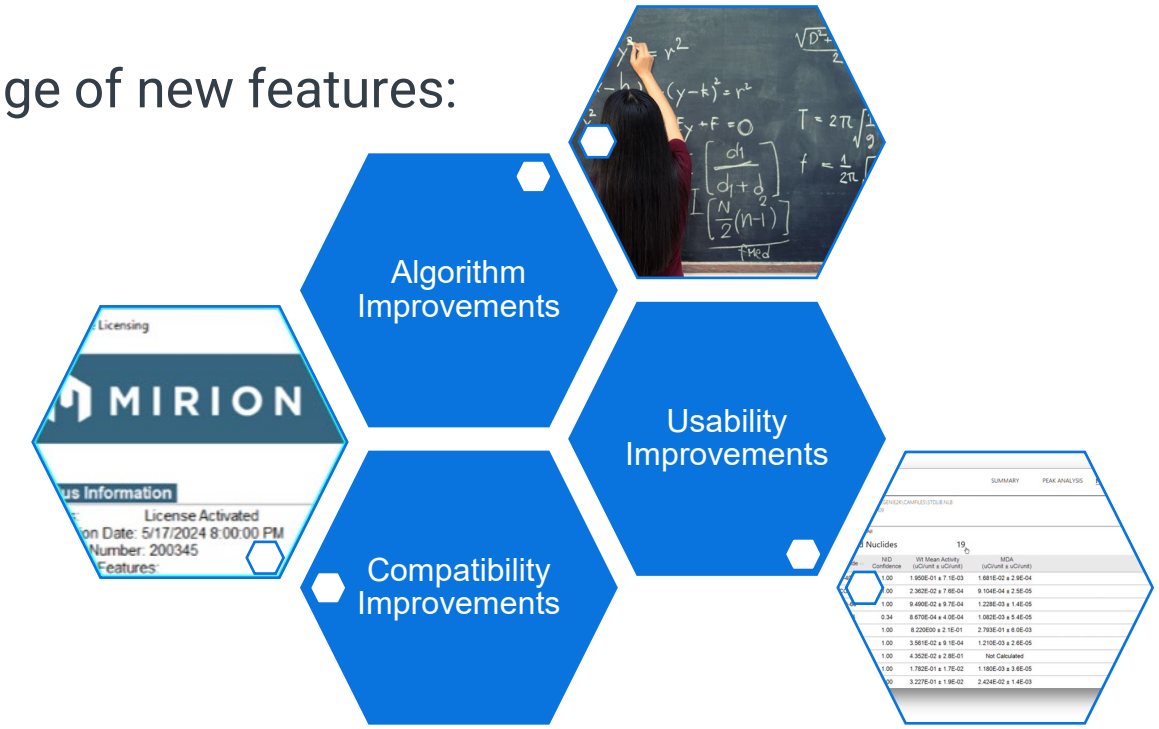
Helsinki

MIRION TECHNOLOGIES

Genie 4.0 Introduction

- Interactive Reports
- Improved FWHM Calibration Option
- Support for Correlations in Calibration Standards
- Automation Improvements with Python Scripting
- More Value: ISOCS, Genie-FieldPro, QA/QC
- Electronic Downloads and Licensing

- Genie 2000's successor
  Genie 4.0 provides a range of new features:

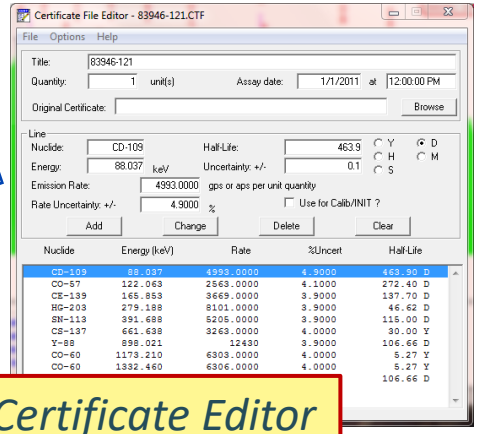


- Focus on Correlation in standards efficiency calibration

Genie 2000 / Genie 4.0

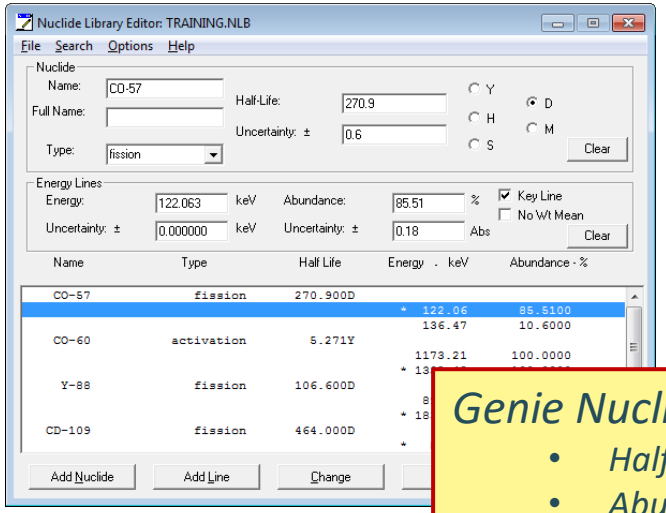
Uncertainty management

- Efficiency calibration Uncertainty
  - Certificate used
    - Rate uncertainty (combine Abundance uncertainty and **Nuclide Activity Uncertainty**)
  - **Net peak area**



Genie Certificate Editor
Activity uncertainty for k=1

- Activity Uncertainty
  - **Net peak area**
  - **Efficiency**
  - Energy line Abundance
  - Nuclide Half life
  - Sample quantity
  - Random (facultative)
  - Systematic (facultative)



Genie Nuclide library Editor
- *Half life uncertainty*
- *Abundance uncertainty*

Genie 2000 /Genie 4.0

Uncertainty management

Activity

- The calculation of the uncertainty on the activity is given by the following formula

$$\sigma_C = C.\sqrt{\left(\frac{\sigma_R}{100}\right)^2 + \left(\frac{\sigma_S}{S}\right)^2 + \left(\frac{\sigma_V}{V}\right)^2 + \left(\frac{\sigma_\varepsilon}{\varepsilon}\right)^2 + \left(\frac{\sigma_y}{y}\right)^2 + \left(\frac{\sigma_K}{K}\right)^2}$$

Where $C = \frac{S}{V.\varepsilon.y.T_1.K_C.K_W}$

σ_R : Random uncertainty
σ_S : Net peak area uncertainty
σ_V : Sample quantity uncertainty
σ_ε : **Efficiency uncertainty**
σ_y : peak Abundance Uncertainty
σ_K : Nuclide Half life Uncertainty

$$C_{A_V} = \frac{\sum_{i=1}^{N} \frac{A_i}{\sigma_{A_i}^2}}{\sum_{i=1}^{N} \frac{1}{\sigma_{A_i}^2}}$$

Formula used for
Weighted mean Activity
Multi-line nuclide

- The uncertainty calculated for the nuclide average activity will be given by the formula

$$\sigma_{A_{moy}} = \sqrt{\frac{1}{\sum_{i=1}^{N} \frac{1}{\sigma_{A_i}^2}}}$$

Genie 2000 / Genie 4.0

Uncertainty management

ISO 11929 MDA

ISO11929 MDA formula, where measurement Uncertainty is part of MDA calculation

ISO11929 formula for decision threshold

$$c_A^* = k_{1-\alpha}.\frac{w}{t_g}\sqrt{n_{b,E} + u^2_{(n_b,E)}}$$

$$w = \frac{1}{P_E.\varepsilon_E.V.f_E}$$

Where:

c_A^*	Decision threshold
$k_{1-\alpha}$	Confidence factor 1.645 for 95%, 1.96 for 97.5%
$n_{b,E}$	Integral of the region
$u^2_{(n_b,E)}$	Integral Uncertainty
t_g	Active count time

ISO11929 MDA formula

$$c_A^{\#} = \frac{2.c_A^* + (k^2.w)/t_g}{1 - k^2.u^2_{rel}(w)}$$

Where:

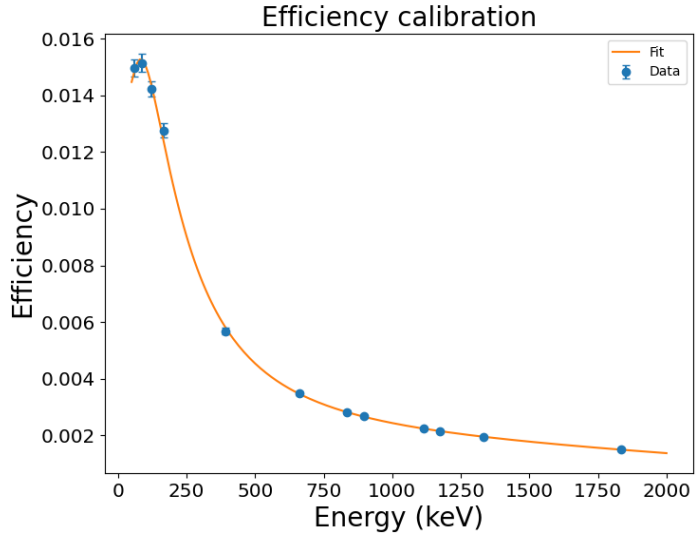
$$u^2_{rel}(w) = \left(\frac{\sigma_P}{P}\right)^2 + \left(\frac{\sigma_{\varepsilon\varepsilon}}{\varepsilon}\right)^2 + \left(\frac{\sigma_V}{V}\right)^2 + \left(\frac{\sigma_f}{f}\right)^2$$

P_E	Nuclide energy line abundance
ε_E	Energy Efficiency
V	Sample quantity
f_E	Decay correction factor

$c_A^{\#}$	Detection Limit (MDA)
c_A^*	Decision thresold
$u^2_{rel}(w)$	Measurement uncertainty

Correlations in Calibration Standards

- Interactive Reports
- Improved FWHM Calibration Option
- **Support for Correlations in Calibration Standards**
- Automation Improvements with Python Scripting
- More Value: ISOCS, Genie-Fieldpro, QA/QC
- Electronic Download and Licensing

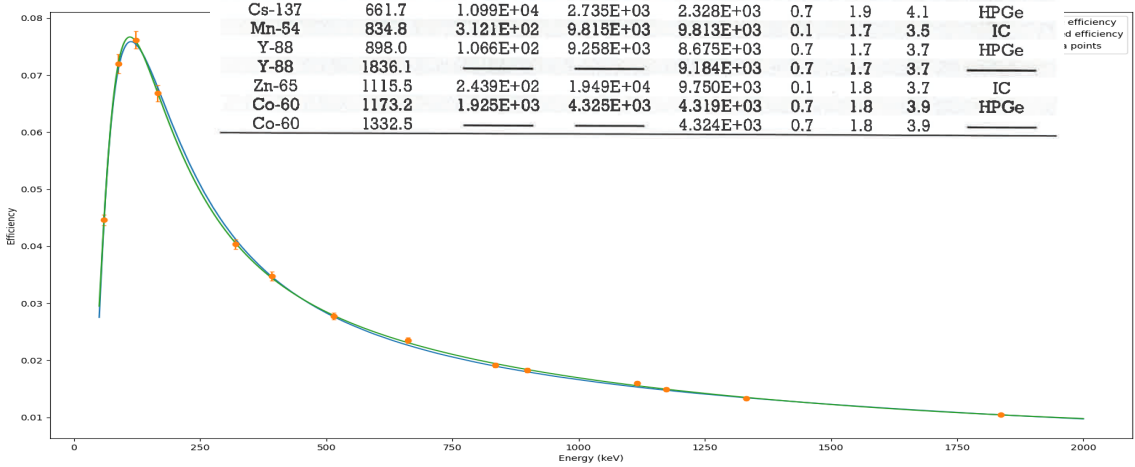


- The uncertainty in an efficiency calibration is propagated to the uncertainty of the unknown sample activity.
- Uncertainty propagation is different for correlated values than independent values:
  - Generally additional independent measurements improve the knowledge of the detector efficiency response, driving down the final activity uncertainty.
  - Additional fully-correlated measurements *do not* improve the knowledge of the detector efficiency response, and the final activity uncertainty may be higher than with values treated independently.
- Uncertainty propagation and management is a topic of focus by regulators and the scientific community

Correlations in Calibration Standards

- Interactive Reports
- Improved FWHM Calibration Option
- **Support for Correlations in Calibration Standards**
- Automation Improvements with Python Scripting
- More Value: ISOCS, Genie-Fieldpro, QA/QC
- Electronic Download and Licensing

Isotope	Gamma-Ray Energy, keV	Half-Life, d	Activity, Bq	Flux, s^{-1}	Uncertainty u_A, %	u_B, %	U, %*	Calibration Method**
Am-241	59.5	1.580E+05	1.356E+04	4.868E+03	0.1	1.8	3.7	4π LS
Cd-109	88.0	4.614E+02	9.354E+04	3.461E+03	0.5	2.0	4.2	HPGe
Co-57	122.1	2.717E+02	2.163E+03	1.851E+03	0.4	1.7	3.5	HPGe
Ce-139	165.9	1.376E+02	3.247E+03	2.597E+03	0.4	1.8	3.7	HPGe
Cr-51	320.1	2.770E+01	9.352E+04	9.268E+03	0.1	1.8	3.7	IC
Sn-113	391.7	1.151E+02	5.726E+03	3.720E+03	0.4	2.0	4.1	HPGe
Sr-85	514.0	6.485E+01	7.275E+03	7.166E+03	0.1	1.8	3.7	IC
Cs-137	661.7	1.099E+04	2.735E+03	2.328E+03	0.7	1.9	4.1	HPGe
Mn-54	834.8	3.121E+02	9.815E+03	9.813E+03	0.1	1.7	3.5	IC
Y-88	898.0	1.066E+02	9.258E+03	8.675E+03	0.7	1.7	3.7	HPGe
Y-88	1836.1	———	———	9.184E+03	0.7	1.7	3.7	———
Zn-65	1115.5	2.439E+02	1.949E+04	9.750E+03	0.1	1.8	3.7	IC
Co-60	1173.2	1.925E+03	4.325E+03	4.319E+03	0.7	1.8	3.9	HPGe
Co-60	1332.5	———	———	4.324E+03	0.7	1.8	3.9	———



Definitions:

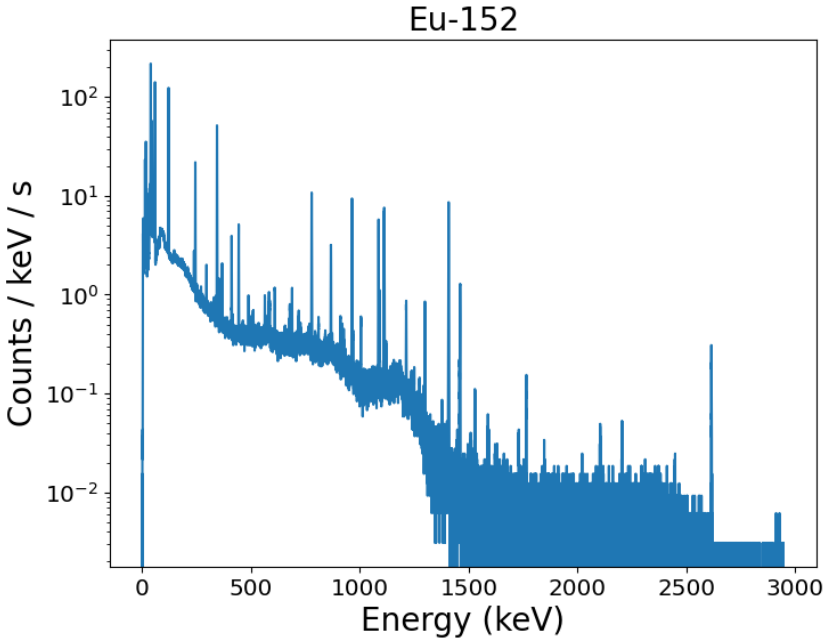
- Two vales are “correlated” if there is a relationship between the measurements
- Two values are independent if there is no information gained about one measurement from the other measurement

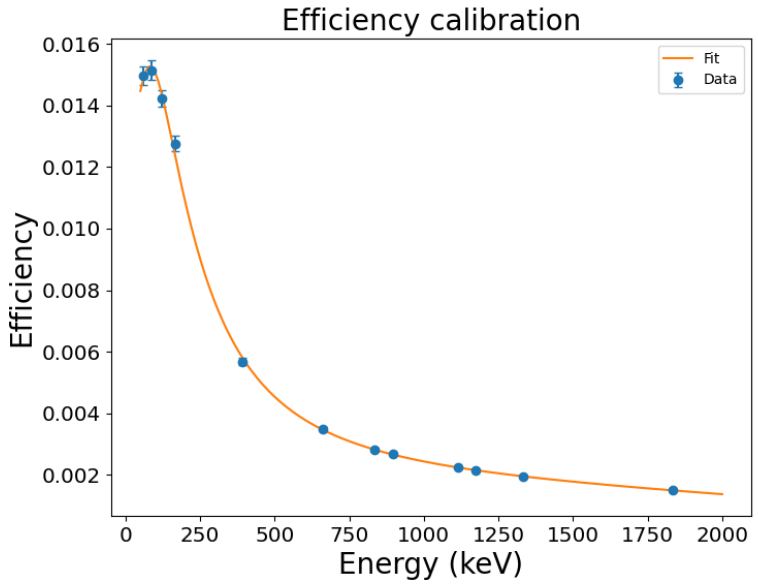
In our application:

- In Gamma Spectroscopy and Efficiency Calibrations, the item being “measured” is the true efficiency of the detector at various energies.
- The measurement values are the calculated “efficiency triplets” from the emission lines of the calibration standard.

Genie 4.0 Correlation improvement Introduction

- For high precision gamma spectrometry, the uncertainty from efficiency calibration is often dominating
- Quantities in gamma spectrometry can be determined using multiple measurements
  - Efficiency as a function of Energy, weighted mean activity of radionuclides
- When these measurements are independent the uncertainty is reduced when more measurements are added
- This leads to uncertainties from the efficiency fit to be lower than the uncertainty of the individual points unless correlations between data points are accounted for



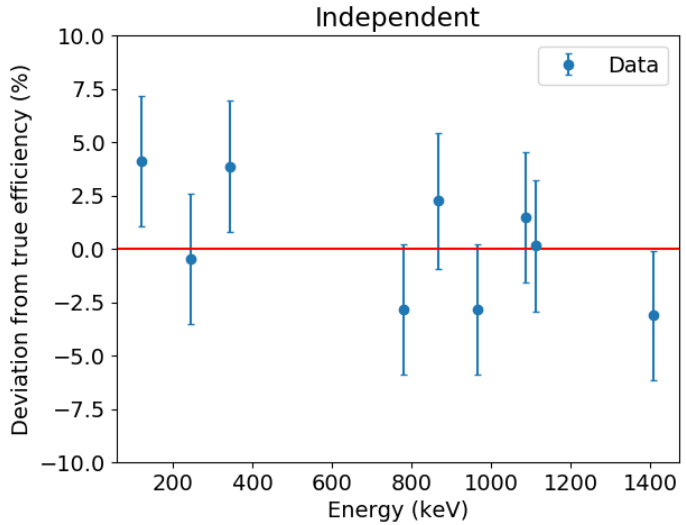


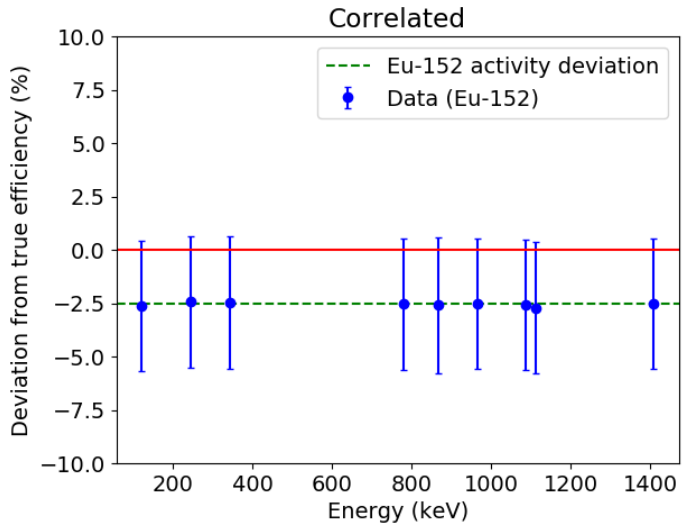
Genie 4.0 Correlation

- Correlations are relations in data that varies together in a way not expected from chance alone
- The emission rate for lines from a multiline radionuclide
- Changes the least squares fit

$$\chi^2 = \sum_{i=1}^{n} \frac{(y_i - f(x))^2}{\sigma_{y_i}^2} \rightarrow \chi^2 = (y - f(x))^T V^{-1} (y - f(x))$$

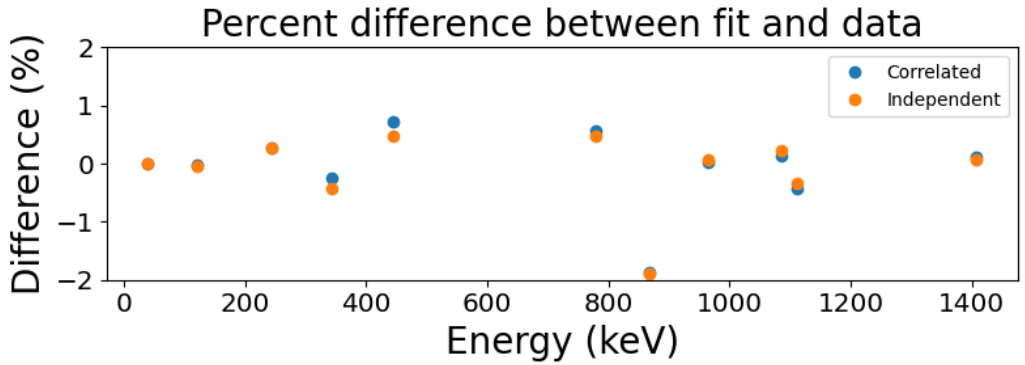
- This has consequences for efficiency calibration, weighted mean activity and interference correction of activities*
- But also for MDA ISO11929 calculation

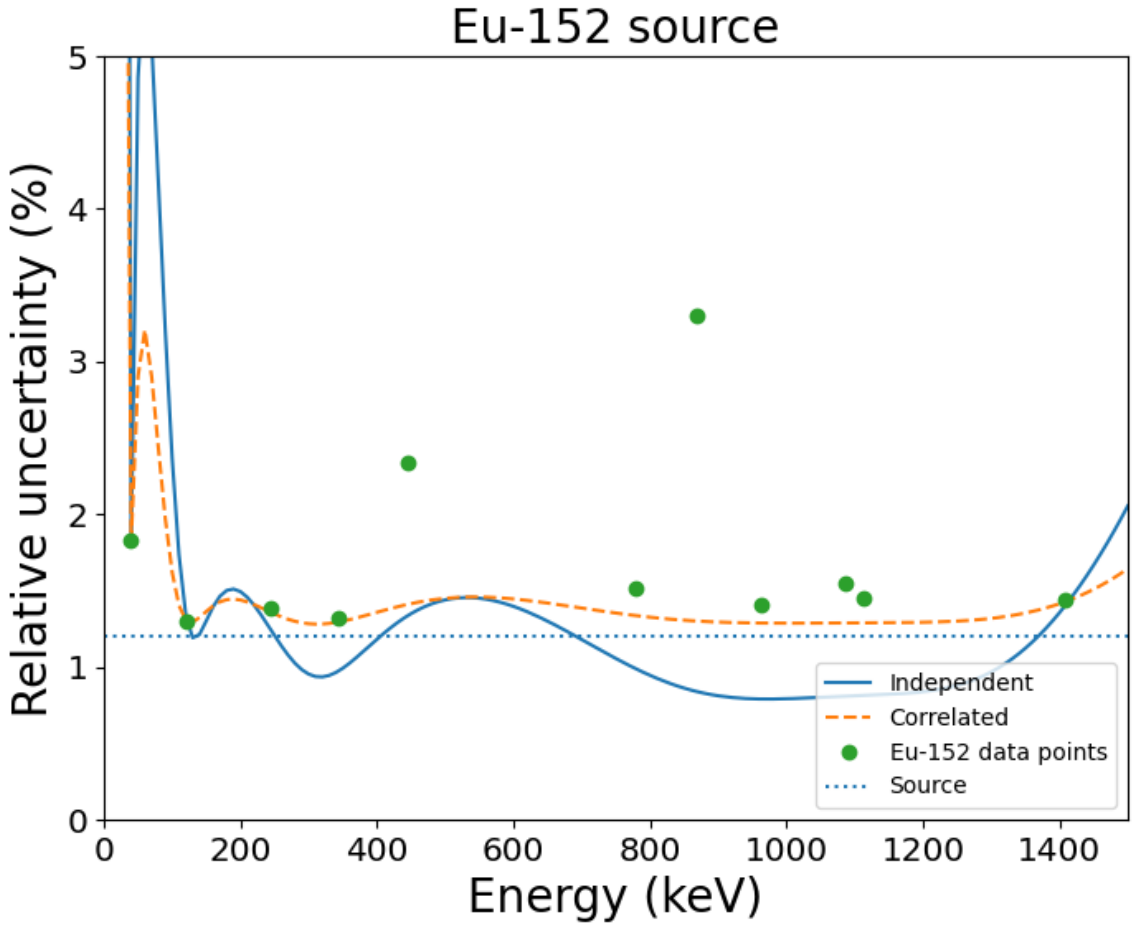




Example 1 - Single radionuclide calibration

- Eu-152 calibration
  - Point source at 30 cm
  - 20% rel eff p-type detector
  - Multiple emission lines with correlation from the radionuclide activity
- Activity has a relative uncertainty of 3.1% at 99% CL

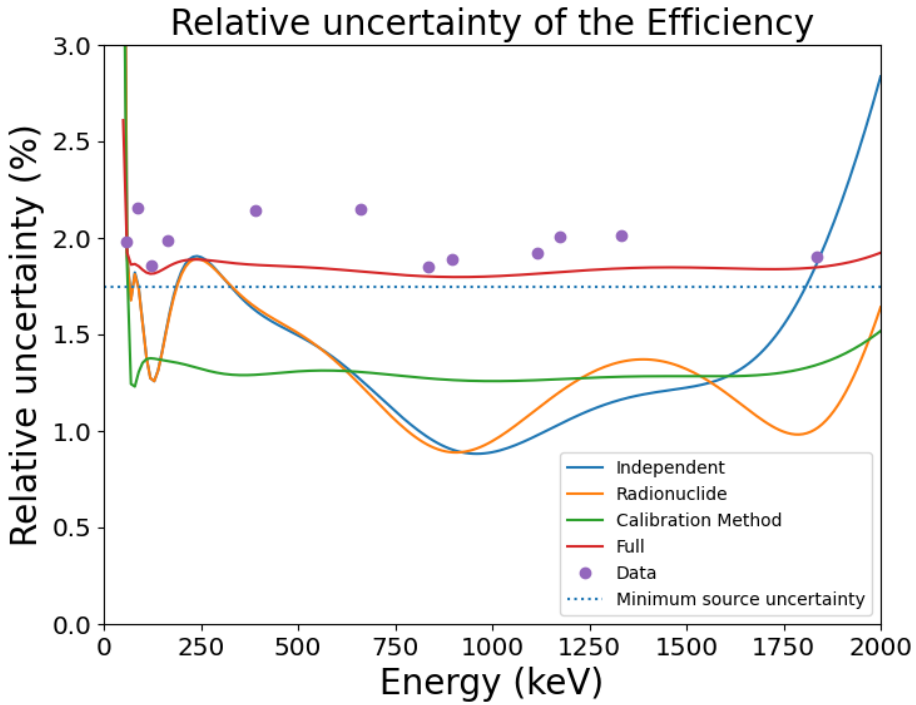


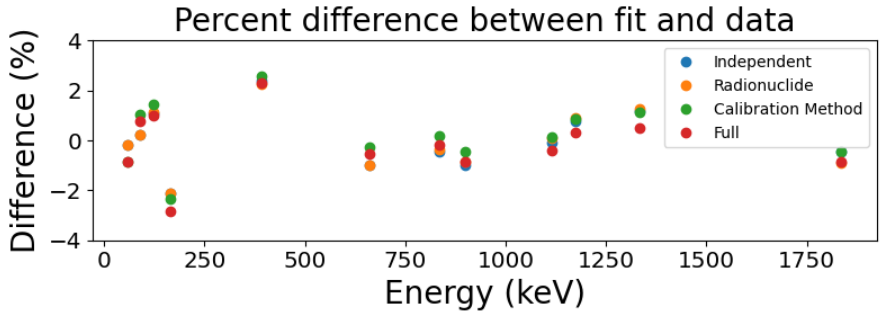


Example 2 - Mixed gamma source

- Multi-radionuclide calibration source
- The level of correlation depends on the assumptions
  - All points independent
  - Points from same radionuclide correlated
  - Points using the same calibration method correlated
  - All points correlated
- For the most conservative assumption the relative uncertainty will be slightly above the lowest relative uncertainty from the certificate.

Isotope	Gamma-Ray Energy, keV	Half-Life, d	Activity, Bq	Flux, s^{-1}	Uncertainty u_A, %	u_B, %	U, %*	Calibration Method**
Am-241	59.5	1.580E+05	1.358E+04	4.875E+03	0.1	1.8	3.7	4π LS
Cd-109	88.0	4.614E+02	9.367E+04	3.466E+03	0.5	2.0	4.2	HPGe
Co-57	122.1	2.717E+02	2.165E+03	1.854E+03	0.4	1.7	3.5	HPGe
Ce-139	165.9	1.376E+02	3.251E+03	2.601E+03	0.4	1.8	3.7	HPGe
Cr-51	320.1	2.770E+01	9.365E+04	9.281E+03	0.1	1.8	3.7	IC
Sn-113	391.7	1.151E+02	5.733E+03	3.725E+03	0.4	2.0	4.1	HPGe
Sr-85	514.0	6.485E+01	7.285E+03	7.175E+03	0.1	1.8	3.7	IC
Cs-137	661.7	1.099E+04	2.739E+03	2.331E+03	0.7	1.9	4.1	HPGe
Mn-54	834.8	3.121E+02	9.828E+03	9.826E+03	0.1	1.7	3.5	IC
Y-88	898.0	1.066E+02	9.270E+03	8.686E+03	0.7	1.7	3.7	HPGe
Y-88	1836.1	———	———	9.196E+03	0.7	1.7	3.7	———
Zn-65	1115.5	2.439E+02	1.951E+04	9.763E+03	0.1	1.8	3.7	IC
Co-60	1173.2	1.925E+03	4.331E+03	4.324E+03	0.7	1.8	3.9	HPGe
Co-60	1332.5	———	———	4.330E+03	0.7	1.8	3.9	———





Weighted mean activity

- Independent weighted mean

$$A = \frac{\sum \frac{A_i}{\sigma_{A_i}^2}}{\sum \frac{1}{\sigma_{A_i}^2}}, \qquad \sigma_A = \sqrt{\frac{1}{\sum \frac{1}{\sigma_{A_i}^2}}}$$

- New Correlated weighted mean algo

$$A = \frac{\sum_{i,j} V_{ij}^{-1} A_j}{\sum_{i,j} V_{ij}^{-1}}, \qquad \sigma_A = \sqrt{\frac{1}{\sum_{i,j} V_{ij}^{-1}}}$$

From covariance matrix $V = \begin{pmatrix} \frac{\sigma_{\varepsilon_1}^2}{\varepsilon_1^2} & \cdots & V_{1n} \\ \cdots & \cdots & \cdots \\ V_{n1} & \cdots & \frac{\sigma_{\varepsilon_n}^2}{\varepsilon_n^2} \end{pmatrix}$

Energy (keV)	Eff:Corr WtM:Corr	Eff:Corr WtM:Indep	Eff:Indep WtM:Corr	Eff:Indep WtM:Indep
121.8	1.39 %	1.39 %	1.39 %	1.39 %
244.7	1.52 %	1.52 %	1.42 %	1.42 %
344.3	1.41 %	1.41 %	1.12 %	1.12 %
444.0	2.47 %	2.47 %	2.43 %	2.43 %
778.9	1.62 %	1.62 %	1.35 %	1.35 %
867.4	3.33 %	3.33 %	3.18 %	3.18 %
964.1	1.48 %	1.48 %	1.08 %	1.08 %
1085.9	1.62 %	1.62 %	1.27 %	1.27 %
1112.1	1.52 %	1.52 %	1.15 %	1.15 %
1408.0	1.63 %	1.63 %	1.61 %	1.61 %
Wt Mean	1.25 %	0.52 %	0.55 %	0.44 %

Relative uncertainty of Eu-152 source 1.20 %

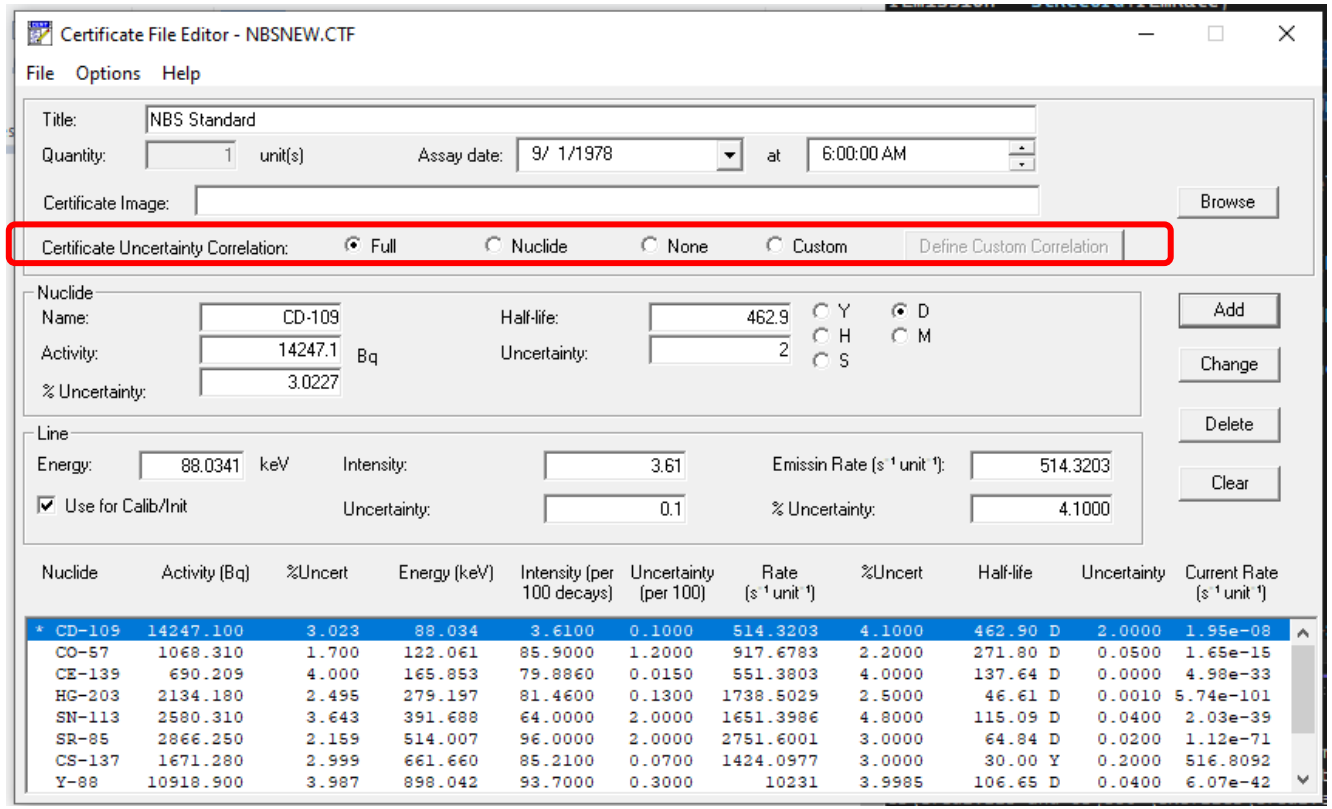
Correlations are supported in Genie 4.0

- **Primary User Focus:** Updated Certificate File Editor (see right).

  This is where the user defines how the calibration source standard emission lines are correlated.

- **Transparent for the user**

  Genie algorithms updated for efficiency calibration fits and nuclide identification with interference correction engine (now called w/ Correlations”) “NID



Conclusion

- Taking correlations into account for efficiency calibration and activity calculations in gamma spectrometry can increase the uncertainty of the radionuclide activities when the efficiency data points are correlated.
- Uncertainties in the independent variable, intensities, are necessary when performing interference correction when the relative uncertainty in the decay data is comparable or large compared to other uncertainties.
- Nuclide Activity Uncertainty Results may increase by 1-3%

Included in Genie 4.0

Genie 4.0 webinar presentation available on our website

[Latest in Gamma Spectroscopy: New Genie™ 4.0 Software (on24.com)]

Thank you

Special thank to our scientist team
Henrik Persson, Troy Anderson, John M Kirkpatrick, Kara Phillips